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STATISTICAL TESTS FOR CLOSURE OF PLATE MOTION CIRCUITS

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Abstract. The standard method of testing whether data describing the relative motion of three plates meeting at a triple junction are consistent with rigid plate tectonics uses the condition of plate circuit closure: the Euler vectors or angular velocities describing the relative motion of each plate pair must sum to zero. Deviations from closure suggest that the data are systematically in error, or that the plate boundaries have been identified incorrectly, or that the plates are non-rigid. We examine two methods, one based on a chi-square test and the other based on an F-ratio test, of testing whether plate motion circuits close. We find the latter test is useful, but the former is of limited value because of the common practice of systematically overestimating errors in plate motion data. We apply the F-ratio test to Minster and Jordan's data for the boundaries dividing the Pacific, Cocos, and Nazca plates, which prove consistent with plate circuit closure. Application of the test to their data for the boundaries dividing the Australian, Antarctic, and African plates shows nonclosure even after deletion of data from the Carlsberg Ridge, which recently has been suggested to divide the African plate from an Indo-Arabian plate. This nonclosure suggests systematic errors in the data or internal deformation of one or more of the plates meeting at the Indian Ocean Triple Junction.

Introduction

The concept of triple junction or plate motion circuit closure [McKenzie and Parker, 1967; Morgan, 1968; LePichon, 1968; McKenzie and Morgan, 1969; Chase, 1972] is central to plate tectonic theory and to the geometrical rigor of plate tectonic analysis based on the assumption that the interiors of plates are rigid when viewed over millions of years. Surprisingly, no rigorous statistical analysis has ever been given for the problem of whether plate motion circuits close. Prior analyses have used subjective evaluations of whether models are consistent with the assumed errors of the data.

For a system of three rigid plates i, j, and k, the Euler vectors or angular velocity vectors obey a 3-plate circuit closure condition

$$ {}_i\omega_j + {}_j\omega_k + {}_k\omega_i = 0 \qquad (1)$$

[McKenzie and Parker, 1967]. A special case occurs at the point $\mathbf{r}_{tj}$, the triple junction where the three plates meet. Since

$$ {}_i\omega_j \times \mathbf{r}_{tj} + {}_j\omega_k \times \mathbf{r}_{tj} + {}_k\omega_i \times \mathbf{r}_{tj} = 0 \qquad (2)$$

follows from equation (1), and is equivalent to

$$ {}_i\mathbf{v}_j + {}_j\mathbf{v}_k + {}_k\mathbf{v}_i = 0 \ ,$$

the three *linear* velocity vectors at the triple junction sum to zero, giving a commonly used closure condition. Although the linear velocity closure condition is always true if the Euler vectors obey closure, linear velocities derived from plate motion data near the triple junction can appear to close even if the Euler vectors derived from more extensive data along the three plate boundaries do not.

In contrast, Euler vectors are typically determined using data along the entire length of the boundaries. Thus, the circuit closure condition using Euler vectors is more powerful because it can reveal misfits occurring anywhere along the boundaries. Systematic misfits when plate closure is enforced may have several different causes including systematic errors in the data, internal deformation of one or more of the plates, or a poorly parameterized model, as might occur if a significant plate boundary was ignored. Rigorous tests of plate motion closure can thus place quantitative limits on intraplate deformation and may provide a useful method for identifying data with possible systematic errors.

In an earlier paper [Stein and Gordon, 1984] we presented statistical methods to test whether the data justified adding an additional plate when the data along a single plate boundary were poorly fit. In this paper we extend this analysis to the problem of plate circuit closure by presenting two tests of closure based on different assumptions about the errors in the data. We discuss the relative merits of the two methods and apply them to synthetic data and to published data describing the motion of the three plates that meet at the Galapagos Triple Junction and the three plates that meet at the Indian Ocean Triple Junction.

The Tests

It is common practice when combining many plate motion data to assign each datum an uncertainty [Chase, 1972, 1978; Minster et al., 1974; Hey et al., 1977; Minster and Jordan, 1978; Tapscott et al., 1980], which is usually intended to equal the standard error of an individual datum (which may really represent an average value determined from several data). These assigned standard errors provide the basis for any decision whether the data are fit within their uncertainties and therefore are consistent with plate circuit closure. A difficulty of assigning errors to these data is that the errors assigned are of necessity subjective. In this paper we consider statistical tests of closure based on two distinct assumptions that could be made about these errors.

The first possible assumption is that these assigned errors are accurate estimates of the true errors in the data, which consist of spreading rates determined from analysis of magnetic profiles crossing spreading ridges, azimuths of transform faults determined by mapping seafloor topography or mapping offsets in magnetic lineations, and orientations of earthquake slip vectors determined from analysis of seismograms. This assumption permits a plate circuit closure test based on a standard chi-square test. This test is equivalent to propagating the errors in the individual best-fitting Euler vectors to test whether the Euler vector sum differs significantly from zero. Unfortunately, the assumption that the errors have been accurately estimated is hard to reconcile with the observation that global plate motion

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models fit the data systematically better than expected from the assigned errors [Minster and Jordan, 1978]. This overestimation of errors is the result of attempting to account for systematic as well as random errors in the data, and has the useful effect of yielding conservative estimates of Euler vector confidence limits.

The second possible assumption is that the *relative* values of the assigned errors are accurate, although they may be systematically too high. This assumption permits a plate circuit closure test based on a standard F-ratio test. We will see that the F-ratio plate circuit closure test is more useful for identifying systematic misfits in the data than the chi-square plate circuit closure test.

Chi-square Test

A widely used statistic for evaluating goodness of fit of a model to N data is χ^2, defined as

$$\sum_{i=1}^{N} \left(\frac{d_i^{obs} - d_i^{pred}}{\sigma_i} \right)^2 \tag{3}$$

where d_i^{pred} is the predicted value of the i^{th} datum, d_i^{obs} is its observed value, and σ_i is the standard error assigned to this observed datum. It is useful to consider the reduced chi-square, χ_ν^2, defined as the ratio of χ^2 to the number of degrees of freedom, $\nu = (N-3p)$, where p is the number of independent Euler vectors (each with three components) estimated from the data. For example, when solving for the best fit Euler vectors for three plates meeting at a triple junction, three parameters give the rotation of the second plate relative to the first, and three more give the rotation of the third relative to the second. The rotation of the third relative to the first needs no additional independent parameters because it can be found by adding the first two Euler vectors. Thus the number of degrees of freedom is the number of data minus six.

Values of χ_ν^2 much greater than 1 suggest poor fits of the model to the data, whereas values of χ_ν^2 near 1 suggest good fits of the model to the data. Reference values of χ_ν^2 for a given number of degrees of freedom are given in standard tables [e.g., Spiegel, 1975]. For this test we compare the experimentally determined value of χ_ν^2 to a reference value that has little likelihood, say 1% or 5%, of being exceeded by chance if the plates are rigid, there are no systematic errors in the data, and the errors have been estimated accurately. A 5% risk level would be appropriate if we expected to study only a single plate circuit, as there is only a 1 in 20 chance of concluding the circuit did not close when it really did. Because a global relative plate motion model contains about fifteen major three-plate circuits, an apparent deviation from rigidity at this risk level might occur although all plates were rigid. We thus use a more conservative 1% risk level.

Values of χ_ν^2 significantly less than 1 do not necessarily imply superior fits of the model to the data. As an example, consider Minster and Jordan's [1978] global dataset with 330 data and 11 plates, where $\nu = 330 - 30 = 300$ and $\chi_\nu^2 = 0.36$. This value of χ^2 is suspiciously small, since values nearer to 1 are expected. There is less than a 1% probability that χ_ν^2 would be less than 0.78, or exceed 1.25. Minster and Jordan [1978] thus inferred that the errors they estimated for their data were systematically too high by a factor of about $\sqrt{3}$. We shall see that such overestimation poses difficulties for the use of χ^2 tests of plate circuit closure.

F-ratio Test

The F-ratio test is a standard statistical test used to compare variances of distributions. Whereas the chi-square test of plate circuit closure described above focuses on the consistency of the final best-fit estimates of relative Euler vectors with the *a priori* assigned errors, the F-ratio test of plate circuit closure described here focuses on the *differences* in the overall fit of two different models (Figure 1) to the data. One model consists of the three Euler vectors derived by simultaneously fitting all the data. Since closure is enforced and the Euler vectors must sum to zero, six independent parameters are determined from the data. The second model consists of the three best fitting Euler vectors derived by separately fitting the data for each plate pair. Since no closure is enforced, the three best fitting vectors need not sum to zero and nine independent parameters are determined from the data.

The test is formulated using χ^2, and is analogous to the test of additional terms widely used in curve fitting. If the values of σ_i are consistently overestimated by a constant multiplicative factor, the value of F is unaffected, which is a potential advantage over the chi-square test. χ^2 determined with 9 adjustable parameters (N-9 degrees of freedom) is always less than χ^2 determined from the same data but with only 6 adjustable parameters (N-6 degrees of freedom). To test if the reduction in χ^2 is greater than would be expected simply because additional model parameters were added, the statistic

$$F_{3,N-9} = \frac{\left(\chi^2(6) - \chi^2(9) \right) / 3}{\chi^2(9) \,/\, (N-9)} \tag{4}$$

is used. This statistic is expected to be F distributed with 3 versus N-9 degrees of freedom [Bevington, 1969]. The experimentally determined value of F is compared to a reference value from tables [e.g., Spiegel, 1975] of $F_{3,N-9}$ with less than a 1% probability of being exceeded by

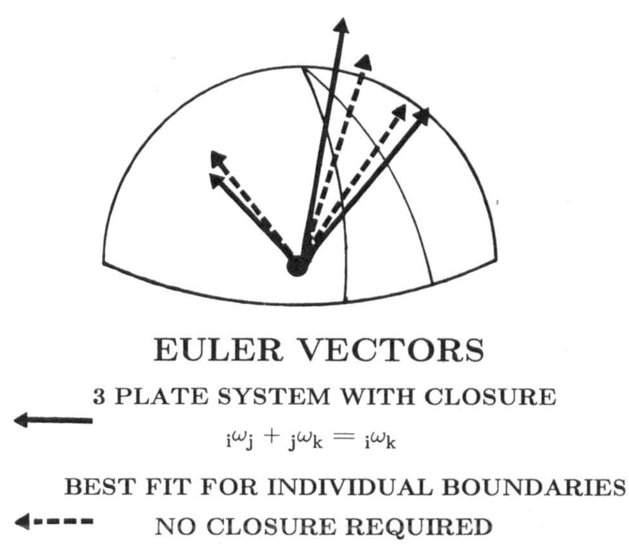

Fig. 1. Geometric constraints on the Euler vectors for a three plate system. The three Euler vectors derived subject to a closure constraint are required to sum to zero, and thus lie in a plane. The three Euler vectors derived from the best fit to each plate boundary independently are not required to obey closure. Given errors in the data, they need not sum to zero or lie in a plane.

chance. If the experimental value exceeds the reference value, then there is a 99% probability that closure is violated.

Application to Synthetic Data

We applied our tests to 200 artificial data sets produced in the following manner. Using Euler vectors obeying closure derived from real data for the relative motions of the Cocos, Nazca, and Pacific plates, ideal spreading rates and azimuths were predicted at the locations of the data. In each realization Gaussian distributed errors from a pseudorandom number generator were added to the ideal data, the standard deviation for the synthetic errors being set equal to that assigned to the corresponding real datum.

Application of both the chi-square and F-ratio tests to each of these sets of synthetic data yielded results in reasonable agreement with our expectations. From the chi-square test, 0 and 5 of the synthetic datasets gave values of χ^2 too large at the 1% and 5% risk levels, respectively. From the F-ratio test, 1 and 13 of the synthetic datasets gave values of F too large at the 1% and 5% risk levels, respectively. These results suggest that both tests give reasonable results when the data errors are accurately estimated by the assigned errors.

We repeated both tests with the same 200 synthetic datasets, but with the σ_i multiplied by $\sqrt{3}$ to examine the usefulness of the tests when the assigned errors are systematically overestimated. As in the test with accurate values of σ_i, we found from the F-ratio test that 1 and 13 of the synthetic datasets gave values of F too large at the 1% and 5% risk levels, respectively. However, from the chi-square test, none of the synthetic datasets gave values of χ^2 too large at the 1% risk level. Moreover, all χ^2 values were so small that every one was less than a reference value that should exceed the observed values only 1% of the time. These results suggest that the F-ratio test gives reasonable results when the data errors are systematically overestimated, but the chi-square test does not.

Application to the Galapagos Plate Motion Circuit

We applied our methods to Minster and Jordan's [1978] data for two different plate circuits, the Galapagos Triple Junction, previously found to close [Hey et al., 1977; Minster and Jordan, 1978], and the Indian Ocean Triple Junction, previously found to fail closure [Minster and Jordan, 1978]. The first of these, the Galapagos Triple Junction, is where the Nazca, Cocos, and Pacific plates meet. Spreading rates, transform fault trends, and earthquake slip vector orientations are well sampled along all three boundaries. The values of χ^2, number of data, and F-ratios are given in Table 1.

Chi-square Test

For the three plate system the value of χ_ν^2 is 0.31 with 39 degrees of freedom (45 data minus 6 adjustable parameters). Tables show that χ_ν^2 would have to exceed 1.6 for the data to be worse fit than expected at the 1% risk level, at face value consistent with closure of this plate circuit. However, χ_ν^2 has less than a 1% probability of being less than 0.55. The small value of χ^2 is thus suspicious, and does not necessarily reflect the internal rigidity of the plates. Instead it seems likely that the errors assigned to the data are systematically too large. We therefore conclude that the application of the chi-square test is not useful here.

TABLE 1: Plate Circuit Statistics

Data Set	χ^2	χ^2_ν	Number of data	Degrees of freedom
		Galapagos		
Cocos Nazca	5.87	0.53	14	11
Cocos Pacific	0.67	0.08	11	8
Nazca Pacific	4.47	0.26	20	17
3-Plate Closure	12.02	0.31	45	39
$F = 1.1$			$F_{.01} = 4.4$	$F_{.05} = 2.9$
		Indian Ocean		
		Minster and Jordan geometry		
Indo-Austr. Africa	4.59	0.33	17	14
Antarctica Africa	2.37	0.12	23	20
Antarctica Indo-Austr.	4.53	0.24	22	19
3-Plate Closure	18.69	0.33	62	56
$F = 11.1$			$F_{.01} = 4.2$	$F_{.05} = 2.8$
		Wiens et al. geometry		
Australia Africa	0.86	0.10	12	9
Antarctica Africa	2.37	0.12	23	20
Antarctica Australia	4.53	0.24	22	19
3-Plate Closure	11.34	0.22	57	51
$F = 7.4$			$F_{.01} = 4.2$	$F_{.05} = 2.8$

F-ratio Test

Table 1 gives values of χ^2 determined assuming closure and values of χ^2 and N (number of data) for the Euler vectors that separately best fit the data from each plate pair. The value of F is 1.1, less than 4.4, which $F_{3,36}$ has only a 1% chance of exceeding if the plates are rigid and there are no systematic errors in the data. It thus seems reasonable to attribute the improvement in fit with 3 instead of 2 Euler vectors merely to that expected from adding insignificant parameters. Thus the data are consistent with plate circuit closure and rigid plate tectonics.

Application to the Indian Ocean Plate Motion Circuit

Based on residual plots, Minster and Jordan [1978] concluded that the motion of the three plates (in their model the Antarctic, Indo-Australian, and African plates) meeting at the Indian Ocean Triple Junction did not close. We apply closure tests to these data along the three plate boundaries (the Southwest Indian Ridge, the Southeast Indian Ridge, and the Central Indian and Carlsberg ridges). Spreading rates, transform fault azimuths, and earthquake slip vector orientations are reasonably well sampled along all three boundaries.

Chi-square Test

For the three plate system the value of χ_ν^2 is 0.33 with 56 degrees of freedom (62 data minus 6 adjustable parameters). Tables show that χ_ν^2 would have to exceed 1.5 for the data to be worse fit than expected at the 1% risk level, again at face value consistent with closure of this plate circuit. However, χ_ν^2 has only a 1% probability of being less than 0.61 by chance. As we found for the Galapagos Triple Junction, the value of chi-square is unreasonably small, suggesting that the test is not useful, presumably because the errors are overestimated. We thus think it inappropriate to conclude from this test that the Indian Ocean plate circuit data obey closure.

F-ratio Test

Table 1 gives values of χ^2 for the Euler vectors that best fit the Indian Ocean data from each plate pair separately. The value of F is 11.1, greater than 4.2, which $F_{3,53}$ has only a 1% probability of exceeding by chance. Thus the F-ratio test shows that the data are fit significantly better by separately determined best-fit Euler vectors than when closure is enforced. We conclude that data from at least one plate pair is systematically misfit when closure is enforced. Minster and Jordan [1978] pointed out that the data along the Southeast Indian Ridge are systematically misfit, and hypothesized that this misfit is caused by internal deformation of the Indian plate.

Wiens et al. [1985] proposed that the Arabian seafloor north of the Carlsberg Ridge and the seafloor north of ~2°N near the Central Indian Ridge move with the Arabian plate and not with the Australian plate. We thus deleted data along the Carlsberg Ridge and north of 2°N along the Central Indian Ridge and repeated our analysis to see how reconfiguration of the boundary affects closure. The value of F, 7.4, although smaller than in the analysis above, is still greater than 4.2, which $F_{3,48}$ has only a 1% probability of exceeding by chance. We conclude that the data are systematically misfit when closure is enforced, although the revised plate geometry decreases the non-closure. Whereas the F-test suggests nonclosure, it cannot distinguish between intraplate deformation and systematic errors in the data. More data will presumably be needed to discriminate between these possibilities.

Conclusions

The chi-square test of plate circuit closure is of little value because of the common practice of overestimating the errors of plate motion data. On the other hand, the F-ratio test of closure seems to be insensitive to systematic overestimation of errors, and provides a simple and useful test of the internal consistency of data in plate circuits. Application of the F-ratio test to Minster and Jordan's [1978] data suggests that the relative motions of the Pacific, Nazca, and Cocos plates are consistent with negligible systematic errors and with rigid plate tectonics, whereas the relative motions of the Australian, African, and Antarctic plates are not. Caution seems justified in attributing systematic misfits identified by the F-ratio test to intra-plate deformation because systematic misfits may reflect systematic errors in the data or a faulty parameterization of the model, such as overlooked plate boundaries.

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