THE DISTRIBUTION OF SEEPAGE WITHIN LAKEBEDS

By M. S. McBRIDE and H. O. PFANNKUCH,¹

Saint Paul and Minneapolis, Minn.

Abstract.—The mutual exchange of water between lakes and contiguous permeable ground-water bodies, which are thin relative to the diameter of the lakes, was modeled digitally. A significant rate of seepage was found to extend only a relatively short distance from shore, thus forming a narrow band around the lake’s perimeter. This near-shore concentration of seepage is an effect only of the geometry of the ground-water flow system, which is governed by the geometry of the body of permeable material, the spatial distribution of permeability within it, and the form of the water table. Near-shore seepage occurs independently of the presence of fine-grained, low-permeability sediments along the bottom materials in the central part of the lake. Digital modeling indicates that the velocity of seepage generally decreases as an exponential function of distance from shore. Field measurements of seepage rates through the bottom of Lake Salle, west-central Minnesota, confirm the model results by demonstrating that both the near-shore seepage band and the exponential decrease in seepage velocity actually exist.

Recent field studies (Allred and others, 1971; Sloan, 1970, 1979; and Shjello, 1968) have demonstrated significant interchange between lakes and ground-water bodies in hydrogeologic situations where such interchange might intuitively seem unlikely. From these studies and others, we are convinced that lakes, rather than being isolated from ground-water bodies by lake-bottom sediments, are generally in close connection with them, forming integral parts of dynamic ground-water flow systems.

This paper deals with seepage in the sense of water moving across the interface between a porous medium and open water or air. Its purpose is to consider how such seepage is distributed over the bottoms of natural lakes; that is, to determine where the rate of seepage is greatest and where least. We determined this first by use of simple but realistic mathematical models and second by field measurements of seepage rates.

The present studies apply to situations in which the width of the lake is greater than, or at least comparable to, the thickness of permeable material underlying the lake. This is generally the situation in Minnesota and Wisconsin, where the glacial drift is seldom more than 150 m thick and where most lakes are greater than 150 m wide. In many places, moreover, a low-permeability layer within the drift may form an effective base for the near-surface ground-water flow, and the thickness of permeable material beneath the lake may be considered to be much less than the total thickness of the drift. Freeze and Witherspoon (1967) have shown that a layer only one order of magnitude lower in permeability than the surface layer can form an effective lower limit to a near-surface ground-water flow system.

FORMULATION OF MATHEMATICAL MODELS

The relation between ground-water flow systems and lakes is shown diagrammatically in figure 1. Flow systems are bounded above by the water table and below by low-permeability materials. Ground-water flow diverges from ground-water divides, converges on some lakes, and diverges from others. Many lakes probably receive inward seepage through part of their beds and discharge water to the ground through another part.

To evaluate the distribution of seepage through a lake’s bottom, we first calculated the distribution of head within the permeable materials near the lake and then calculated the rate of seepage to or from the lake, as a function of position on the lake bottom, by Darcy’s law.

The modeling approach used assumes the following:

1. Ground-water flow is two dimensional, as would occur if the lake shores and ground-water divides were straight, parallel, and infinitely long.

2. The principal axes of permeability are everywhere aligned with the coordinate axes; that is, maximum permeability is always in the horizontal direction and minimum permeability in the vertical direction.

¹Department of Geology and Geophysics, University of Minnesota.
3. While layers may be present with permeability differing from layer to layer, within each layer the permeability is everywhere the same.

4. Position of the water table is constant. The model is thus steady state.

5. The permeable materials are thin in comparison to the width of the lake.

6. Under steady-state conditions, the distribution of head conforms to a generalized form of Richards’ equation (Richards, 1931):

$$\frac{\partial}{\partial x} \left[ K_h(x,\phi) \frac{\partial \phi}{\partial x} \right] + \frac{\partial}{\partial z} \left[ K_s(x,\phi) \frac{\partial \phi}{\partial z} \right] = 0 \quad (1)$$

in which \( \phi \) = hydraulic head, defined as the height above some arbitrary level to which water will rise in a tube whose lower opening is at the point where head is to be specified,

- \( K_h(x,\phi) \) = hydraulic conductivity at point \((x,\phi)\) in the \(x\)-direction, and
- \( K_s(x,\phi) \) = hydraulic conductivity at point \((x,\phi)\) in the \(z\)-direction.

This equation can be solved to give head as a function of \(x\) and \(z\) within a region, if boundary conditions are specified along the region’s edges. The boundary conditions imposed are shown in figure 1, in which two appropriate regions for modeling are bounded by dashed lines. Along the water table, the head (that is, the altitude of the water table) is known from field observations or is assumed for hypothetical problems. Along the bottom of each region, an impermeable boundary is assumed.

Side boundaries are not so easy to define. Places where ground-water flow diverges, as at ground-water divides, or converges, as at the center of the middle lake in figure 1, are modeled as impermeable (imaginary) boundaries. As a first approximation, we have assumed such boundaries to be straight and vertical. Beneath a lake both fed from and feeding a ground-water body, there must be an equipotential line having the same head as the lake. In situations where the lower boundary is close to horizontal, this equipotential line is nearly vertical. (In the absence of field data it may be modeled as being at the center of the lake.) The modeling program was constructed so that any vertical boundary could be made either impermeable or constant-head.

In this study, Richards’ equation was solved numerically, using the point successive over-relaxation method, details of which may be found in standard texts (von Rosenberg, 1969; Mitchell, 1969). The mathematical methods are not new, being based on the work of Freeze and Witherspoon (1969).

**SEEPAGE AS PREDICTED BY MODELS**

The results obtained from the models may best be understood by examining one model in detail. Figure 2A illustrates a model with isotropic porous materials, horizontal lower boundary, and impermeable side boundaries. Numbered lines are equipotential lines; the units of head are arbitrary.

Except for equipotential 10 which is affected by vertical flow near the divide, the equipotential lines are nearly vertical; thus, ground-water flow is nearly horizontal through most of the section.

Figure 2B represents the left third of figure 2A, with the vertical exaggeration adjusted so that flow lines orthogonal to the equipotential lines may be shown in greater detail near the lakeshore. Note in figure 2B that flow lines that emerge beneath the lake are concentrated close to shore, where most seepage enters in a narrow band, rather than being distributed uniformly over the entire bottom.

Concentrated seepage below lake level may be quantitatively evaluated by determining the relation between seepage rate and distance from shore. The numerical method used in this study yields values of head at a finite number of nodal points, which cover the region of interest in a regular rectangular grid. Therefore, in figure 2B, imagine one column of nodes to the right of the lake edge and an adjacent column to the left. The flux from a node in the right column to the adjacent node in the left column is the difference in heads divided by the horizontal distance, multiplied by the vertical distance and by the horizontal hydraulic conductivity. This is simply an application of Darcy’s law. Fluxes so calculated for each pair of nodes along the columns are summed to give the total horizontal flux between the two columns.

Horizontal fluxes can be calculated in the same way between all pairs of nodal columns lying beneath the lake. Fluxes so calculated are plotted against distance from shore on a logarithmic grid in figure 2C. A straight line seems to fit the data satisfactorily, implying that flux decreases exponentially with distance.
That is, the flux can be represented approximately as a function of distance from shore by an equation of the form \( f = f_0 e^{-cX} \), where \( f \) is the total flux across a vertical line through the aquifer at a distance \( X \) from shore, \( f_0 \) the flux at the shoreline, and \( -c \) is the slope of the graph of flux against distance on semilogarithmic coordinates, as shown in figure 2C. However, seepage into or from the lake bed is of primary interest, rather than horizontal ground-water flux in the aquifer. The seepage into or from the lake bed per unit width of lake bed is equal to the rate of change of the horizontal flux with distance, since change in flux implies a loss or gain in flow which can occur only by seepage through the lake bed. In equation form,

\[
\frac{d}{dX} \left( f_0 e^{-cX} \right) = -cf_0 e^{-cX}
\]

where \( I \) is the seepage rate per unit width of lake bed and the term \(-cf_0 \) is equal to the seepage rate at the shoreline.

These equations show that a plot of seepage rate per width of lake bed against distance from shore, on semilogarithmic coordinates, will be a straight line having the same slope as a plot of horizontal ground-water flux against distance from shore. If the flux and seepage terms are plotted in dimensionless form, by dividing each flux by the flux at the shoreline, \( f_0 \), and each seepage rate by the seepage rate at the shoreline, \(-cf_0\), the two plots become identical. In equation form,

\[
\frac{f}{f_0} = \frac{I}{I_0} = e^{-cX}.
\]

Thus, a plot of horizontal flux in dimensionless form against distance from shore, such as that shown in figure 2C, can be taken equivalently as a plot of seepage rate in dimensionless form against distance from shore.

Ten model runs for selected parameter values are illustrated in figure 3 to show the effect of the shape of the water table and subsurface hydraulic conductivity variations on the near-shore concentration of seepage. In these illustrations, the vertical scale has been exaggerated about 15 times. However, because the ratio of horizontal to vertical hydraulic conductivity has been assumed equal to 10 for each model, the vertical exaggeration for equivalent isotropic models would be about 5 times. Because of this scale distortion, vertical flow components may be pronounced than might be anticipated. Data points represent flux values on the graphs accompanying the models, but the straight lines through them can equally well be taken to represent seepage. Table 1 summarizes hydraulic conductivities of the various layers and boundary conditions in the models, as depicted in figures 2 and 3.
Figure 3.—Vertical-section models having various boundary and internal conditions. Low and high indicate degree of permeability.
Table 1.—Parameters used in theoretical models

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<th>Figure No.</th>
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<th>Smaller layer</th>
<th>Boundary conditions</th>
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Figure 3A is identical to figure 2, except that the depth of the section is greater. Note that the seepage rate in figure 3A falls off more slowly with distance than in figure 2A because of greater transmissivity.

Figure 3B and 3C differ only in the forms of their water tables. In figure 3B the water table is a parabola and in figure 3C a straight line. The graphs of seepage are similar. Evidently here the concentration of seepage is controlled by a constant-head boundary at the lake bottom, rather than by the form of the water table away from the lake.

Figures 3D, 3E, and 3F illustrate the effect of low-permeability bottom sediments. In D, such sediments are absent. In E, they cover the central two-thirds of the lakebed and in F, the entire bed. The seepage graphs for D and E coincide over most of their lengths. Over 95 percent of the seepage enters the lakes in the regions where the graphs are coincident. The end of the graph in E turns down sharply, evidently affected by the edge of the sediments. F shows a considerably different result. The sharp bending of the equipotential lines near shore, where they pass from underlying high-permeability materials into low-permeability lake sediments, reveals that flow is much more nearly vertical in the lake sediments than in the material below. In effect, low-permeability sediments spread out nearshore where the seepage under more of the lake by slowing flow into the lake. The result is clear from the seepage graph for F whose slope is only half as great as those shown in D and E.

Figure 3G shows a lake connected with a sloping water table. Water flows into the lake on the right side and out on the left. Note that the water table was sketched so that the horizontal flux toward the lake was greater than that away from it. Both in and out fluxes were made dimensionless by dividing them by the flux toward the lake, so the two graphs are displaced from one another. In a real lake, a difference between ground-water fluxes toward and away from the lake could be accounted for by losses from the lake by streamflow or evaporation. These components of a lake's water balance are not explicitly included in this model.

Little of the ground-water flow passes beneath the lake. Instead, most is diverted upward as it approaches the lake, passes through the lake, moves downward, and eventually comes back to the horizontal. The graph of flux toward the lake shows that at 10 nodes from shore, or 40 percent of the way across the lake, the horizontal ground-water flux is 0.04, or only 4 percent of its value at the lake shore. Thus, 96 percent of the ground water has entered the lake prior to this point. The same situation occurs on the outflow side of the lake. The lake’s depth is 15 percent of the thickness of the underlying materials and its width about 110 percent if the materials are considered isotropic. These figures are reasonable for a real lake. We believe that it is common for most ground water flowing in the vicinity of a lake to be diverted through the lake, because real lakes are commonly several aquifer thicknesses in width.

Figures 3H, I, and J have the same water table and lower boundary as figure 3D, but differ in having inhomogeneous permeable materials beneath the lakes. In H, the high-permeability layer at the bottom of the section serves to spread seepage somewhat more evenly across the lake bottom than in D. This is reflected in a smaller slope of the graph in H. A similar effect is shown in I, even though the high-permeability layer is less extensive, covering less than half the bottom of the section. In J, the high-permeability material forms a lens and lies much closer to the lake bottom than in H and I. Evidently, this lens has a strong effect on the distribution of seepage, because here the decrease in seepage rate is linearly related to the distance from shore rather than exponentially, as in all the other figures. Note that the graph's grid is linear rather than semilogarithmic.

It may be concluded from these theoretical models that ground-water inflow is concentrated near the shores of lakes under a wide variety of hydrogeologic conditions and that in many places seepage rate will decrease exponentially with distance from shore.

**APPLICABILITY OF MODELS TO FIELD SITUATIONS**

The models presented in figures 2 and 3, of course, do not cover all hydrogeologic situations. We believe, however, that the conclusions drawn from the models are applicable to a wide variety of lakes for the following reasons:

First, the model sections are nondimensional. No linear dimensions enter into Richard's equation (eq 1) or into any of the boundary conditions; thus, a given...
model may represent either a large field section or a small one. Figure 2, for example, can represent a lake a few tens of metres in diameter lying in permeable materials a few metres thick, or it can represent a lake several kilometres across lying in material hundreds of metres thick.

Second, the distribution of potential does not depend on magnitudes of permeabilities of the porous materials. This means in nonhomogeneous sections the forms of the equipotentials are governed by relative permeabilities of the various layers rather than by absolute values.

Third, each model may represent a variety of anisotropies and finite-difference grids. The horizontal flux between adjacent nodes whose heads differ by \( \Delta \phi \) is given by Darcy’s law as:

\[
Q_x = K_x \Delta \phi \Delta x
\]

where \( K_x \) = horizontal hydraulic conductivity,

\( \Delta x \) = grid spacing in vertical direction, and

\( \Delta \phi \) = grid spacing in horizontal direction.

The variables characterizing permeability and the grid geometry may be isolated in a single constant,

\[
C_x = K_x \frac{\Delta \phi}{\Delta x}
\]

Similarly, for the vertical flux

\[
Q_z = K_z \Delta \phi \Delta z
\]

Each node of the model may, therefore, be characterized by a value of \( C_x : C_z \), rather than by particular values of \( K_x, K_z, \Delta x \), and \( \Delta z \). A given model can represent any combination of permeabilities and node spacings for which, at each node, the term \( \frac{K_x (\Delta z)^2}{K_z (\Delta x)^2} \) is equal to the value of \( C_x : C_z \) characterizing that node.

In summary, the anisotropy can be varied as long as the ratio of coordinate spacings is varied in a compensating way. In all the models presented above, the ratio \( \frac{\Delta z}{\Delta x} \) was set at 0.1 and the anisotropy \( \frac{K_x}{K_z} \) at 10 for the largest layer; thus \( \frac{C_x}{C_z} \) was 0.1.

Situations certainly exist in which seepage is not concentrated along lake shores. An example is where the width of a lake is very small compared to the thickness of the underlying ground-water flow system. Freeze and Witherspoon (1966) have presented models of this condition in which the water table is undulating. These could represent regions of hummocky topography, where ground-water bodies are recharged in topographic highs and discharged by evaporation from small ponds in intervening depressions. The ponds have diameters much less than the thickness of the sections. The models show flow lines rising almost vertically to some of the depressions. Evidently, the seepage occurs almost uniformly across the bottoms of these depressions; thus, the concentration noted previously is absent.

The geologic possibilities are too diverse for a general rule to apply; but we may venture, on the basis of the models available, that near-shore concentration of seepage will be significant only where a lake’s diameter is at least roughly comparable to the thickness of the underlying permeable materials.

**FIELD MEASUREMENTS OF SEEPAGE RATES**

Conclusions drawn from theoretical models are much more convincing when supported by field observations. Such observations of seepage were provided by Lee (1972) as part of a study of Lake Sallie, Becker County, west-central Minnesota. Lake Sallie has an area of 5.2 km² and a mean depth of 6 m. It is kidney shaped, 3.3 km long and 1.7 km wide, and lies in sandy glacial outwash which has an average thickness of 15 m and a hydraulic conductivity of about 0.1 cm/s. The outwash is underlain by a thick layer of clay till having a hydraulic conductivity of \( 3.5 \times 10^{-8} \) cm/s. Ground-water movement in the outwash is toward the lake along almost all of the lake’s perimeter. Fourteen percent of the inflow to the lake is ground-water seepage.

Lee made several hundred measurements of the rate of seepage into Lake Sallie at distances from 3 to 110 m from shore. The seepage meter used is illustrated in figure 4. The end of a steel drum was cut off to form a shallow metal collector pan that was imbedded, open side down, a few centimetres into the sand on the lake bottom. A thin-walled plastic bag, containing a weighed amount of water but no air, was attached to the tube. After a few hours or days, the bag was reweighed. Any water that seeped upward through the area of lake bottom covered by the pan was trapped.

![Diagram](Figure 4.—Operation of a seepage meter.)
and diverted into the plastic bag. The change in weight was converted to an equivalent volume of seepage which, when divided by the area of the pan and time for which the bag was connected, gave the seepage velocity.

A representative selection of the data is plotted in figure 5A through 5F. Data in each graph come from measurements made close to a straight line extending perpendicularly from shore out into the lake. All lines of measurements are along a nearly straight part of the southwest shore of Lake Sallie. The distance from line A, on the north to line F, on the south, is about 800 m. As much as possible the data in each graph are from a single month. When more than one measurement was made at a location, the results were averaged; thus, most data points in figure 5 represent two or more measurements.

Although there is much scatter, a straight line seems to fit the data as a satisfactory first approximation. Thus, the seepage rate decreases as an exponential function of distance from shore in the field examples, just as it did in the theoretical cross sections previously presented.

**WIDTH OF THE ZONE OF RAPID SEEPAGE**

The width of the zone of rapid seepage in a real lake was investigated using field data from Lake Sallie. Consider figure 6A; a strip of lake bottom 1 μm wide extends perpendicularly from shore into the lake. The seepage velocity, S, is a function of the distance from shore, X, for points along the strip.

At Lake Sallie (fig. 6) the seepage velocity decreases one order of magnitude for every 60 m (6 × 10^5 μm) of distance, approximately. The seepage velocity is about 0.6 μm/s at the shoreline. Here then, log S = -(X/6 × 10^5) + log 0.6, or S = 0.6 (10^-(X/6 × 10^5)), where S is expressed in micrometres per second and X in micrometres. The total seepage rate from the strip, between the shore and a specific distance L from the shore, is given by

\[ Q(L) = 0.6 \int_{X_{min}}^{X_{max}} 10^{-(X/6 \times 10^5)} dX \]

where X is the variable of integration and L is the upper limit.

Evaluating this integral analytically gives the formula

\[ Q(L) = -1.56 \times 10^7 \left(10^{-\left(L/6 \times 10^5\right)} - 1\right) \]

for total seepage rate as a function of L.

If L is taken at the center of Lake Sallie, about 1 km from shore, seepage through the entire strip amounts to 1.56 × 10^7 μm/s. The percentage of this total taking place within a given distance from shore is shown in figure 6B. One-half of the discharge occurs within 17 m of shore, 90 percent within 60 m, and 99 percent within 120 m. Thus, at Lake Sallie, the width of the zone of high seepage is, roughly, 100 m, or one-tenth the distance to the center of the lake. Note that the edge of the fine-grained sediments in the center of Lake Sallie is 800 m from shore at the location investigated. This again emphasizes the importance of the geometry of the ground-water system and the relatively small role that lake sediments may often have in controlling the distribution of seepage within lakes.

![Figure 5](image)

**Figure 5.** Seepage rates as a function of distance from shore at Lake Sallie.

![Figure 6](image)

**Figure 6.** Seepage from Lake Sallie. A. Strip of lake bottom used in calculating width of zone of rapid seepage. B. Percentage of total seepage occurring within a given distance from shore.
SUMMARY AND CONCLUSIONS

Digital modeling of lake-ground water systems shows that seepage of water into or out of lakes tends to be concentrated near the shore. The seepage rate is greatest at the shore and decreases with increasing distance from shore. In many places, the rate of decrease is exponential. Hydrogeologic conditions exist where the decrease is linear, however, and, no doubt, more complicated situations occur in nature. For seepage concentration, the width of the lake cannot be much less than the thickness of the underlying permeable materials. This condition is not met in all lakes, but probably holds for many, if not most.

Field measurements at Lake Sallie demonstrated the existence of a zone of concentrated seepage near the shoreline. The measurements showed much scatter but indicated plainly that the decrease in seepage rate with distance from shore was approximately exponential. The rate of decrease was one order of magnitude for every 60 m of distance.

The investigations reported here suggest the possibility of using seepage measurements to determine the seepage component of lake-water balances independent of other components. Such a procedure would be cheaper and possibly more accurate than determining seepage as a residual in a water budget.

Because of the narrowness of the zone of rapid seepage, measurements of seepage rate can be confined to a small fraction of the total lakebed. Furthermore, this zone constitutes the shallowest and most accessible part of the lakebed. In many lakes, seepage measurements can be made by wading, rather than from boats or by diving. However, uncertainties must be cleared before this approach can be attempted with confidence.

In particular, the causes of the large amount of scatter in the graphs of seepage rate against distance from shore (fig. 5) are unknown. Some possible causes are that (1) areal distribution of seepage may be irregular, with considerable variations in seepage rates within distances of a few metres, (2) unrecognized defects may exist in the measuring apparatus or in the experimental technique, and (3) seepage rates may fluctuate rapidly with time, perhaps as the result of changes in lake level or barometric pressure.

Another uncertainty concerns distribution of seepage throughout the year. Monthly water balances obtained by Mann and McBride (1972) suggest that almost all seepage into Lake Sallie occurs during March through June. Whether this is common or not is unknown. At present, it is hard to predict how many and how frequent seepage measurements should be made for adequate definition of the total annual seepage in a given lake.

Seepage has received less attention than some other components of lake-water balances. We hope that the results presented here will serve as steps toward obtaining a greater knowledge of seepage mechanisms, both through more detailed field studies and more realistic mathematical models.

REFERENCES CITED


