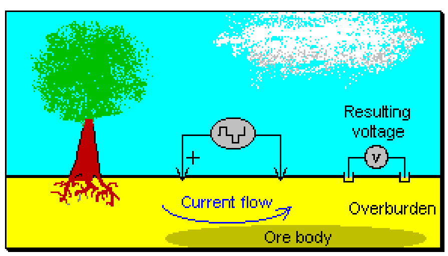
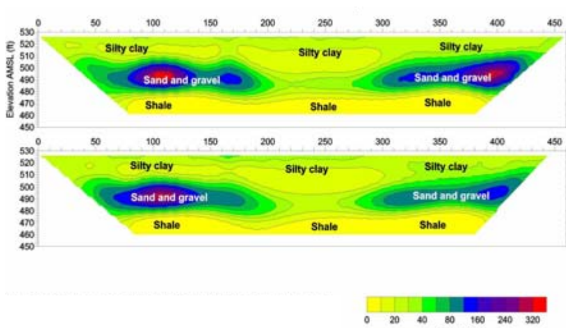
GLE 594: An introduction to applied geophysics

Electrical Resistivity Methods

Fall 2004

Theory and Measurements

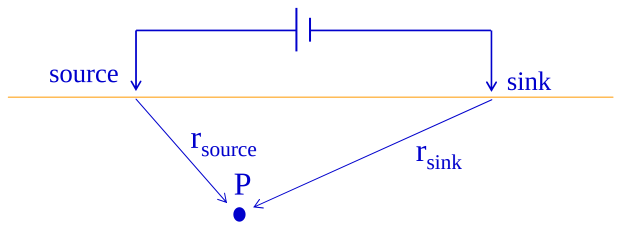




Reading:
Today: 210-223
Next Lecture: 223-251

Two Current Electrodes: Source and Sink

- Why run an electrode to infinity when we can use it?



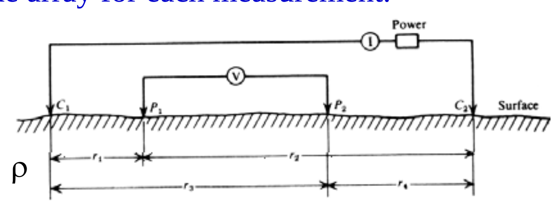
$$V_{source} = \frac{i\rho}{2\pi r_{source}} \qquad V_{sink} = \frac{i\rho}{2\pi r_{sink}}$$

Total Voltage at P:

$$V_p = V_{source} - V_{sink} = \frac{i\rho}{2\pi}\left(\frac{1}{r_{source}} - \frac{1}{r_{sink}}\right)$$

Measurement Practicalities

Can't measure potential at single point unless the other end of our volt meter is at infinity. This is inconvenient. It is easier to measure *potential difference (ΔV)*. This lead to use of four electrode array for each measurement.

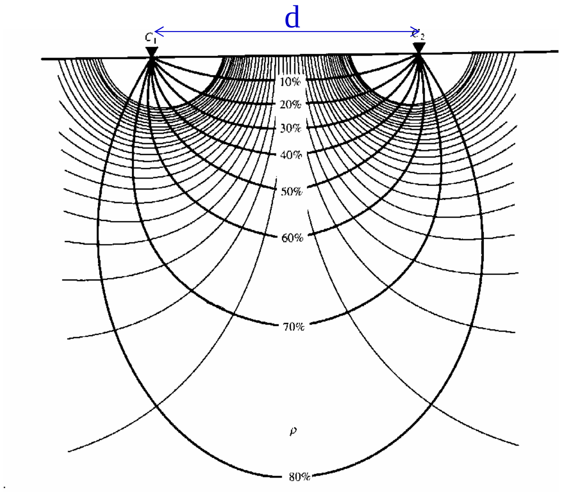


Resulting measurement given as $\Delta V = V_{P1} - V_{P2} = \frac{\rho I}{2\pi}\left(\frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{r_3} + \frac{1}{r_4}\right)$

Can be rewritten $\Delta V = \rho I \frac{G^*}{2\pi}$

where $G^*/2\pi$ is sometimes referred to as the ***Geometrical Factor***

Current density and equipotential lines for a current dipole



fraction total current

$$i_f = \frac{2}{\pi}\tan^{-1}\left(\frac{2z}{d}\right)$$

i_f=0.5 at $z = \frac{d}{2}$

i_f=0.7 at $z = d$

Wider spacing → Deeper currents

Apparent Resistivity

Previous expression can be rearranged in terms of resistivity: $\rho=(\Delta V/I)\ (2\pi/G)$.

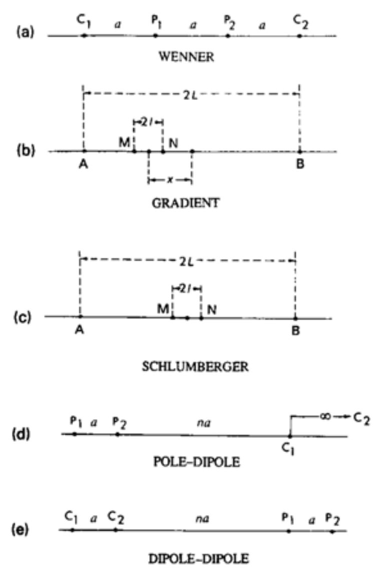
This can be done even when medium is inhomogeneous. Result is then referred to as *Apparent Resistivity*.

ρ_1

ρ_2

Definition:Resistivity of a fictitious homogenous subsurface that would yield the same voltages as the earth over which measurements were actually made.

Geometrical Factors



(a) $$\rho_a = 2\pi a(\Delta V/I)$$

(b) $$\rho_a = \frac{\pi}{I}\frac{(L^2 - x^2)^2}{L^2 + x^2}\frac{\Delta V}{2l}$$

(c) $$\rho_a = \frac{\pi L^2}{I}\frac{\Delta V}{2l}$$

(d) $$\rho_a = 2\pi an(n+1)\frac{\Delta V}{I}$$

(e) $$\rho_a = \pi an(n+1)(n+2)\frac{\Delta V}{I}$$

Array advantages and disadvantages

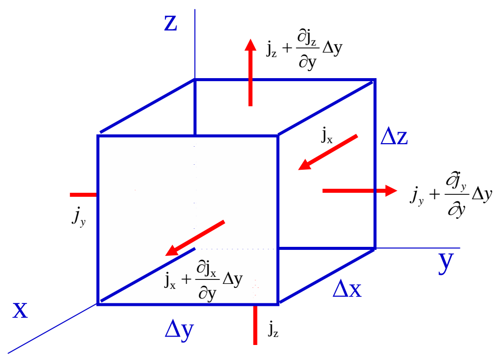
Array	Advantages	Disadvantages
Wenner	1. Easy to calculate ρ_a in the field 2. Less demand on instrument sensivity	1. All electrodes moved each sounding 2. Sensitive to local shallow variations 3. Long cables for large depths
Schlumberger	1. Fewer electrodes to move each sounding 2. Needs shorter potential cables	1. Can be confusing in the field 2. Requires more sensitive equipment 3. Long Current cables
Dipole-Dipole	1. Cables can be shorter for deep soundings	1. Requires large current 2. Requires sensitive instruments

Governing Equation

Continuity: What goes in must comes out

$$\left(j_x - \frac{\partial j_x}{\partial x}\Delta x - j_x\right) + \left(j_y - \frac{\partial j_y}{\partial y}\Delta y - j_y\right) + \left(j_z - \frac{\partial j_z}{\partial z}\Delta z - j_z\right) = 0$$

$$-\frac{\partial j_x}{\partial x}\Delta x - \frac{\partial j_y}{\partial y}\Delta y - \frac{\partial j_z}{\partial z}\Delta z = 0$$



Current Density (like hydro q):

$$\vec{j} = \frac{\vec{i}}{A}$$

Governing Equation

Applying Ohm's Law:

$$j_x = -\frac{1}{\rho}\frac{\partial V}{\partial x};\ j_y = -\frac{1}{\rho}\frac{\partial V}{\partial y};\ j_z = -\frac{1}{\rho}\frac{\partial V}{\partial z}$$

$$\frac{\partial}{\partial x}\left(\frac{1}{\rho}\frac{\partial V}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{1}{\rho}\frac{\partial V}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{1}{\rho}\frac{\partial V}{\partial z}\right) = 0$$

or using

$x = r\cos\theta$, $y = r\sin\theta$, and $x^2 + y^2 = r^2$

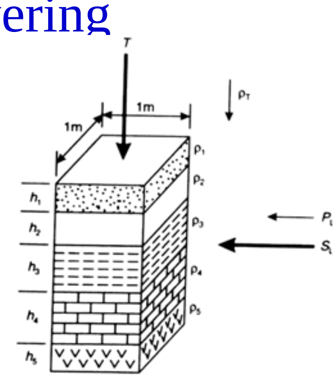
$$\left.\begin{aligned}\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0\\ \frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial z^2} = 0\end{aligned}\right\}\nabla^2 V = 0 \quad \Rightarrow \quad \text{LaPlace's equation}$$

Governing Equation - Solution

- The Laplace's equation is a homogeneous, partial second order differential equation
- Solution:
  - Exact solutions: only for simple geometries
  - Graphical solutions: Flow nets, master charts
  - Numerical solutions: finite difference and finite elements solutions
  - Approximate solutions: methods of fragments
  - Physical analogies (electrical, hydraulic and heat flow)

Geo-electric Layering

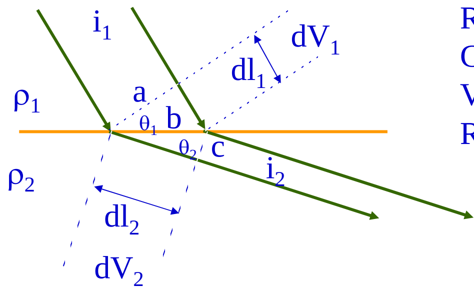
- Often the earth can be simplified within the region of our measurement as consisting of a series of horizontal beds that are infinite in extent.
- Goal of the resistivity survey is then to determine thickness and resistivity of the layers.



Longitudinal conductance (one layer):	$S_L = h/\rho = h\sigma$
Transverse resistance (one layer):	$T = h\rho$
Longitudinal resistivity (one layer):	$\rho_L = h/S$
Transverse resistivity (one layer):	$\rho_T = T/h$
Longitudinal conductance (one layer):	$S_L = \Sigma(h_i/\rho_i)$
Transverse resistance (one layer):	$T = \Sigma(h_i\rho_i)$

Voltage and Flow in Layers

Tangent Law: The electrical current is bent at a boundary



Relations:

Current: $i_1 = i_2$

Voltage: $dV_1 = dV_2$

Resistivity: $\rho_1 > \rho_2$

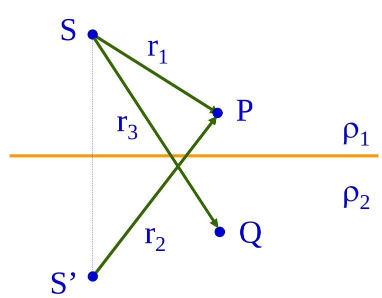
$$\frac{\rho_2}{\rho_1} = \frac{\tan\theta_1}{\tan\theta_2}$$

If $\rho_2 < \rho_1$ then the current lines will be refracted away from the normal

If $\rho_2 > \rho_1$ then the current lines will be refracted closer to the normal

Voltage and Flow in Layers

Method of electrical image



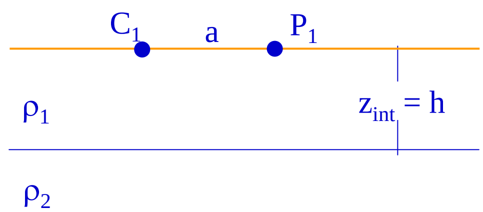
Voltages at points P and Q:

$$V_P = \frac{I\rho_1}{4\pi}\left(\frac{1}{r_1} + \frac{k}{r_2}\right)$$

$$V_Q = \frac{I\rho_2}{4\pi}\left(\frac{1+k}{r_3}\right)$$

where $k = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$

Solving the differential equation for two layers and a source and sink



Governing Equation

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial z^2} = 0$$

Boundary Conditions

1. $i_z = 0\big|_{z=0}$ — No current at surface
2. $V_1 = V_2$ at $z = z_{interface}$ — Voltage is continuous
3. $\frac{1}{\rho_1}\frac{\partial V_1}{\partial z} = \frac{1}{\rho_2}\frac{\partial V_2}{\partial z}$ at $z = z_{interface}$ — Normal current density is continous
4. $V = \frac{i\rho_1}{2\pi\left(r^2 + z^2\right)^{\frac{1}{2}}}$ at $r = 0, z = 0$ — Particular solution

Layer Calculations

- Can use for image theory for multiple boundaries. For two layer case:

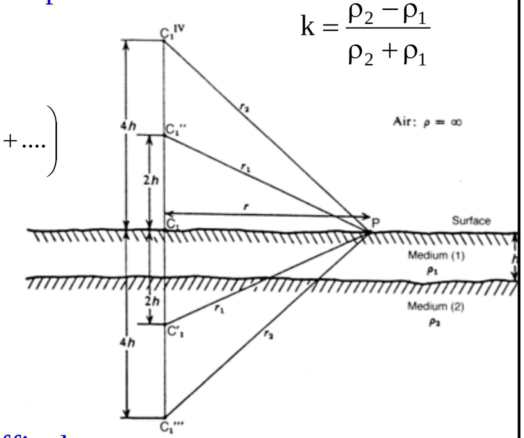
$$k = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$$

$$V_p = \frac{I\rho_1}{2\pi}\left(\frac{1}{r} + \frac{2k}{r_1} + \frac{2k^2}{r_2} + \dots\dots + \frac{2k^n}{r_n} + \dots\dots\right)$$

$$= \frac{I\rho_1}{2\pi}\left(\frac{1}{r} + 2\sum_{n=1}^{\infty}\frac{k^n}{r_n}\right)$$

where

$$r_n = \sqrt{r^2 + (2nh)^2}$$



- It obviously gets much more difficult with more layers.

Layer Calculations (cont.)

- Integral method: $V_p = \frac{I\rho_1}{2\pi}\int_0^\infty K(\lambda)J_0(\lambda r)d\lambda$
- J0 is the *Bessel function of zero order*.
  - *K(λ)* given by relationship $K(\lambda) = \frac{T_1(\lambda)}{\rho_1}$
- *$T_i(\lambda)$* solved for *recursively* upward from bottom layer to layer 1 using:

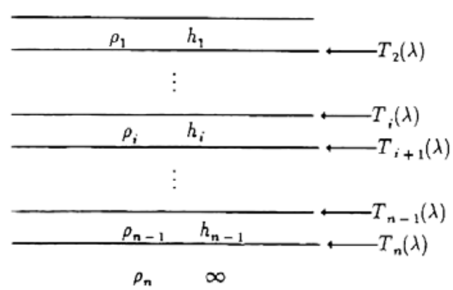
$$T_i(\lambda) = \frac{[T_{i+1} + \rho_i \tanh(\lambda h_i)]}{[1 + T_{i+1}\tanh(\lambda h_i)/\rho_i]}.$$

where

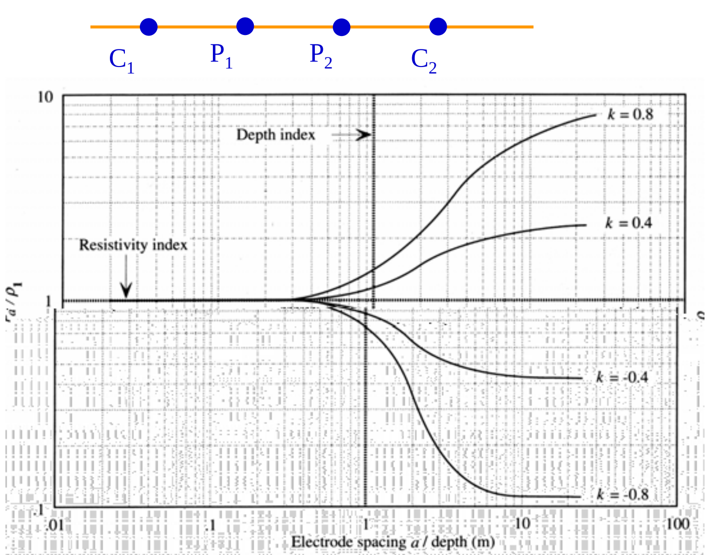
$$\tanh(\lambda h_i) = \frac{e^{2\lambda h_i} - 1}{e^{2\lambda h_i} + 1}.$$

and

$$T_n(\lambda) = \rho_n .$$



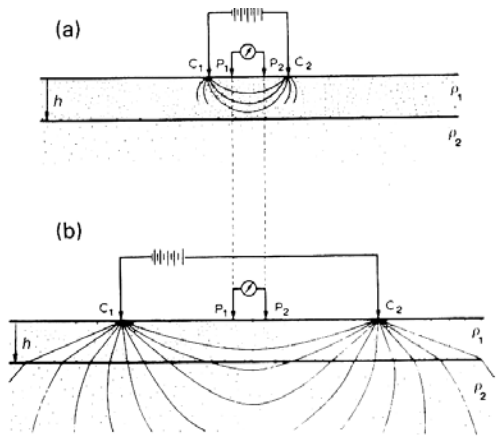
Solutions for a Wenner Array for two layers



$$k = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$$

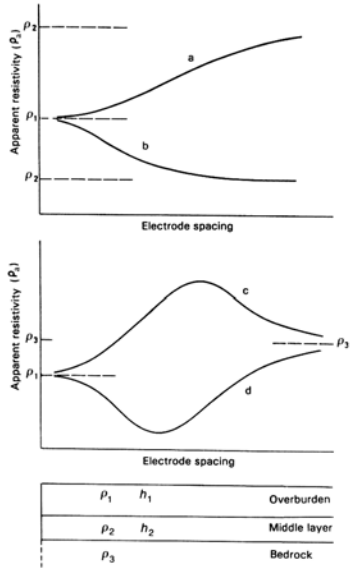
Vertical Electric Sounding

- When trying to probe how resistivity changes with depth, need multiple measurements that each give a different depth sensitivity.
- This is accomplished through *resistivity sounding* where greater electrode separation gives greater depth sensitivity.



VES Data Plotting Convention

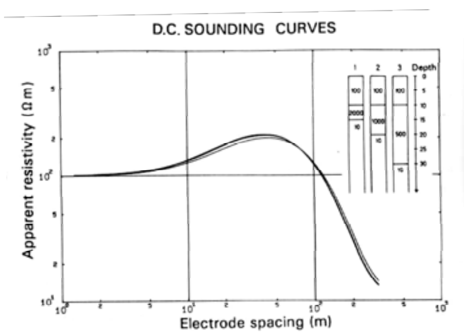
- Plot apparent resistivity as a function of the log of some measure of electrode separation.
  - Wenner – *a spacing*
  - Schlumberger – AB/2
  - Dipole-Dipole – *n spacing*
- Asymptotes:
  - Short spacings << h_1, $\rho_a = \rho_1$.
  - Long spacings >> total thickness of overlying layers, $\rho_a = \rho_n$
- To get $\rho_a = \rho_{true}$ for intermediate layers, layer must be thick relative to depth.



Equivalence: several models produce the same results

- Ambiguity in physics of 1D interpretation such that different layered models basically yield the same response.
- Different Scenarios:
  - Conductive layers between two resistors, where lateral conductance (σ_h) is the same.
  - Resistive layer between two conductors with same transverse resistance (ρ_h).

Equivalence: several models produce the same results



- Although ER cannot determine unique parameters, can determine range of values.
- Also exists in 2D and 3D, but much more difficult to quantify. In these multidimensional cases simply referred to as *non-uniqueness*.

Suppression

- Principle of *suppression:* Thin layers of small resistivity contrast with respect to background will be missed.
- Thin layers of greater resistivity contrast will be detectable, but equivalence limits resolution of boundary depths, etc.