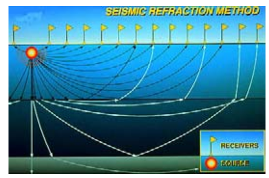
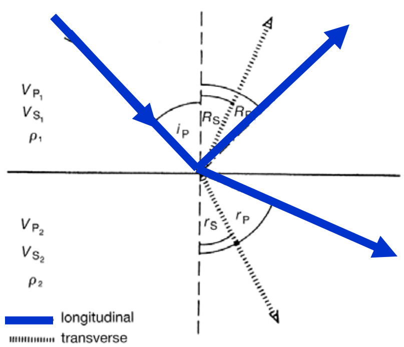
Seismic methods: Refraction I



Refraction reading: Sharma p158 - 186

Pre-Critical incidence

Reflection and refraction

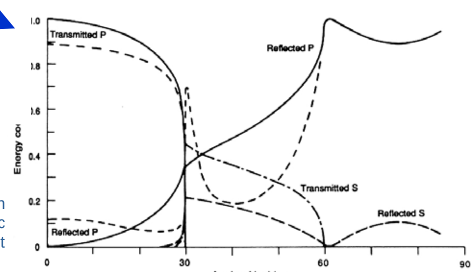


Snell's Law:

$$\frac{\sin i_P}{V_{P1}} = \frac{\sin R_P}{V_{P1}} = \frac{\sin r_P}{V_{P2}} = p$$

where p is the **ray parameter** and is constant along each ray.

Reflection and transmission coefficients for a specific impedance contrast



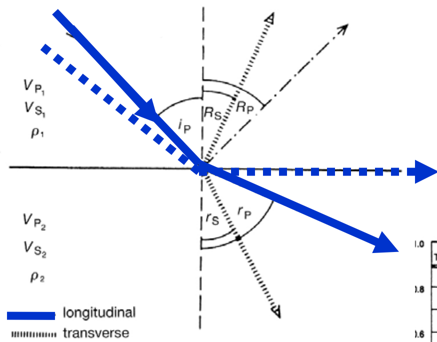
Critical incidence

When r_P = 90° i_P = i_C the critical angle

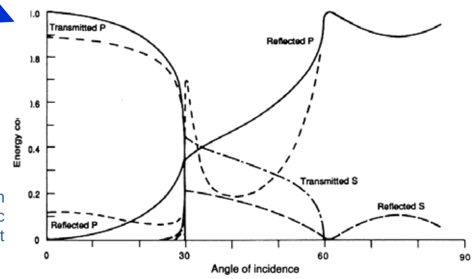
$$\sin i_C = \frac{V_{P1}}{V_{P2}}$$

The critically refracted energy travels along the velocity interface at V_2 continually refracting energy back into the upper medium at an angle i_C

➔ a **head wave**



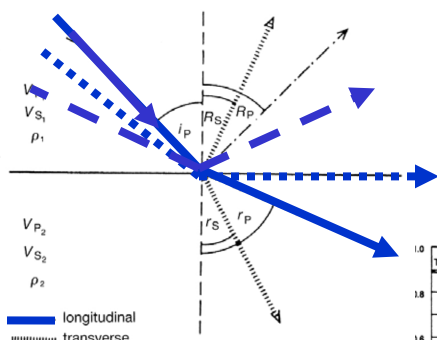
Reflection and transmission coefficients for a specific impedance contrast



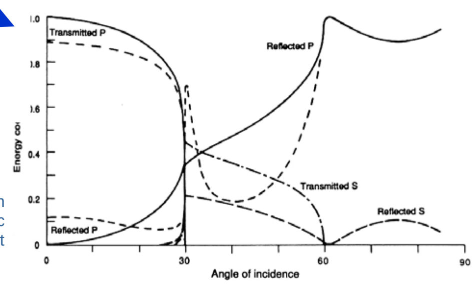
Post-Critical incidence

The angle of incidence > i_C

➔ No transmission, just reflection



Reflection and transmission coefficients for a specific impedance contrast



Horizontal interface

Traveltime equations

Direct wave:

$$T = \frac{x}{V_1}$$

Head wave:

$$T = T_{SB} + T_{DD'} + T_{BD}$$

$$T = \frac{2h_1}{V_1 \cos i_c} + \frac{x - 2h_1 \tan i_c}{V_2}$$

$$T = \frac{x}{V_2} + \frac{2h_1\sqrt{V_2^2 - V_1^2}}{V_2 V_1}$$

T = ax + b

slope: $1/V_2$

intercept: gives h_1

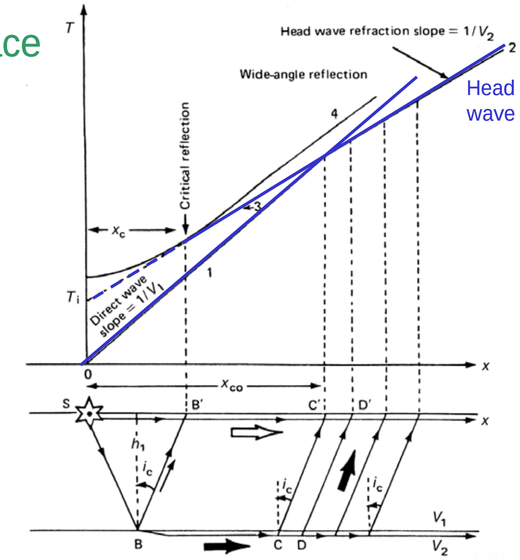


Fig. 4.34 Principle of the seismic refraction method. Travel-time curves for direct waves, critically refracted waves (head waves), late refracted arrivals, and reflected waves are shown by numbers 1–4, respectively. Note that critically refracted waves start arriving after a critical distance x_c, but they overtake the direct waves at a crossover distance x_{co}.

Horizontal interface

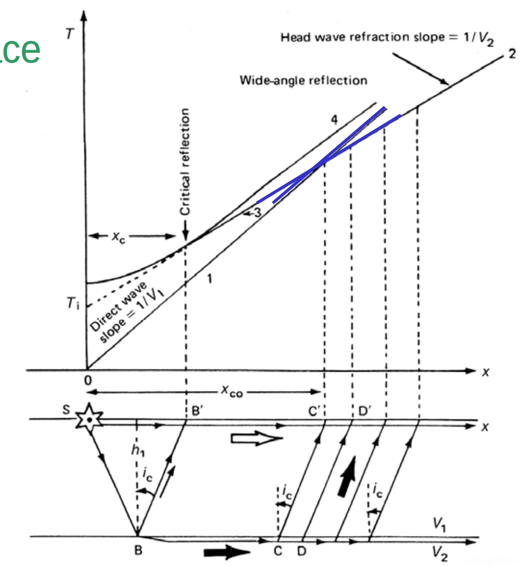
Crossover distance, x_{co}

Where the direct and head wave cross. Their travel times are equal:

$$\frac{x_{co}}{V_1} = \frac{x_{co}}{V_2} + \frac{2h_1\sqrt{V_2^2 - V_1^2}}{V_2 V_1}$$

$$x_{co} = 2h_1 \frac{\sqrt{V_2 + V_1}}{\sqrt{V_2 - V_1}}$$

Another approach to obtaining layer thickness



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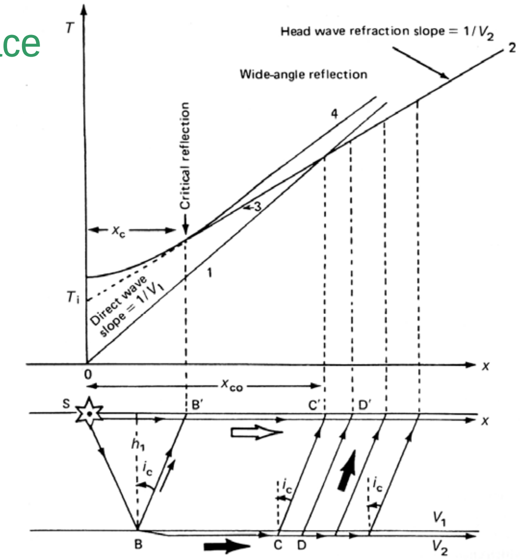
Horizontal interface

Reflections

The **critical reflection** is the closest head wave arrival.

At shorter offsets there are low amplitude reflections (used in reflection seismology).

At greater offsets there are wide-angle reflections.



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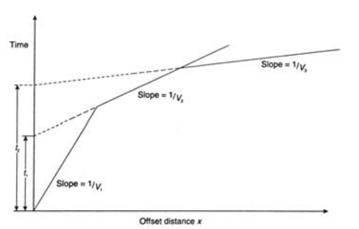
Three-layer model



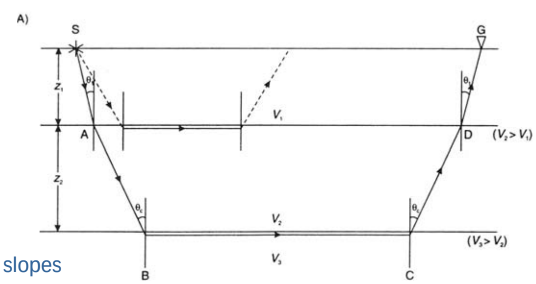
Traveltime

$$T_{SG} = \frac{SA}{V_1} + \frac{AB}{V_2} + \frac{BC}{V_3} + \frac{CD}{V_2} + \frac{DG}{V_1}$$

$$T_{SG} = \frac{2z_1}{V_1 \cos\theta_1} + \frac{2z_2}{V_2 \cos\theta_c} + \frac{x - 2z_1 \tan\theta_1 - 2z_2 \tan\theta_c}{V_3}$$

With some manipulation

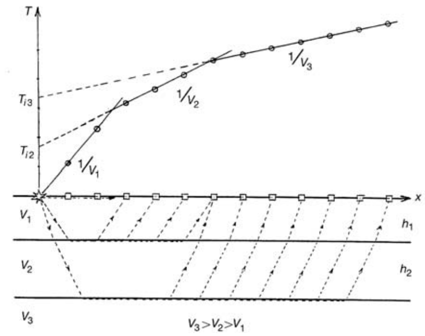
$$T_{SG} = \frac{x}{V_3} + \frac{2z_1\sqrt{V_3^2 - V_1^2}}{V_3 V_1} + \frac{2z_2\sqrt{V_3^2 - V_2^2}}{V_3 V_2}$$



1. Determine V_1, V_2, V_3 from slopes
2. Determine z_1 from 1st intercept
3. Determine z_2 from 2nd intercept

Multiple-layered models

For multiple layered models we can apply the same process to determine layer thickness and velocity sequentially from the top layer to the bottom



Head wave from top of layer 2:

$$T = \frac{x}{V_2} + \frac{2h_1\sqrt{V_2^2 - V_1^2}}{V_2V_1}$$

Head wave from top of layer 3:

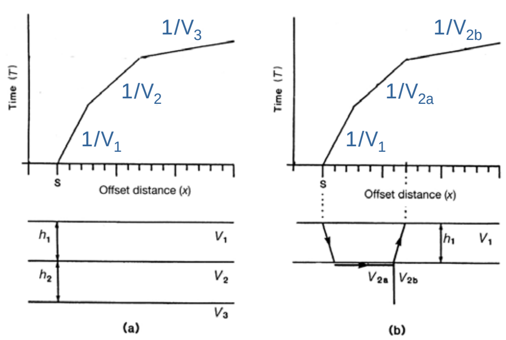
$$T = \frac{x}{V_3} + \frac{2h_1\sqrt{V_3^2 - V_1^2}}{V_3V_1} + \frac{2h_2\sqrt{V_3^2 - V_2^2}}{V_3V_2}$$

Head wave from top of layer n:

$$T = \sum_{j=1}^{n-1}\left(\frac{2h_j\sqrt{V_n^2 - V_j^2}}{V_nV_j}\right) + \frac{x}{V_n}$$

Horizontal vs. vertical velocity contrasts

A three-horizontal layer model can produce the same traveltime curve as a single horizontal layer over a vertical velocity contact



Head wave continues into 2b

Horizontal vs. vertical velocity contrasts

Use a long-offset shot

- Leave the geophones fixed and move shot to greater offset

In horizontal layers case the shape of the traveltime curve is unchanged, just shifted in space.

In vertical velocity contrast case the crossover distance remains fixed but is time shifted.

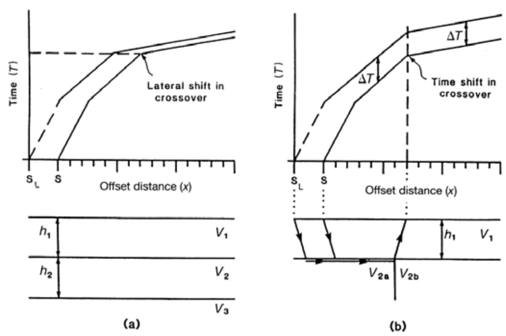


Fig. 4.36 Identical travel-time curves arising from a single shot S over (a) a horizontal three-layer earth, and (b) a two-layer earth with lateral velocity change in the second layer. The ambiguity can be resolved by an additional long offset shot, S_L. Dashed lines on long offset travel-time curves are segments of the travel-time curves that would be observed if geophones were placed between S and S_L. The three-layer case (a) can be distinguished from the two-layer case (b) by noting a lateral shift in the crossover point in the first case and a vertical shift in the crossover point in the second case. (Modified from Lankston, 1990.)

Mapping vertical contacts

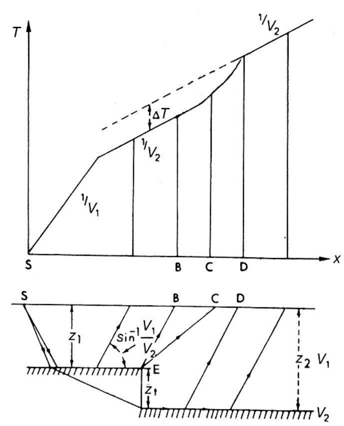
Small offsets

A vertical step causes an offset on the traveltime curve

- The relation of velocity to the slope remains unchanged
- The offset can be calculated from the time offset, ΔT

$$z_t = \frac{\Delta T V_2 V_1}{\sqrt{V_2^2 - V_1^2}}$$

- Diffractions link the two head wave curves
- Depth, z1, is calculated from the intercept in the usual way



Mapping vertical contacts

Infinite/large offsets

For infinite/large vertical offsets there is no secondary head wave

Three segments

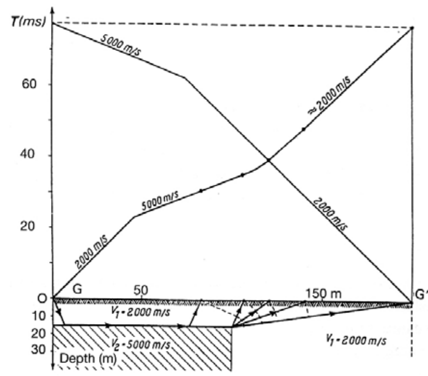
- **Direct wave**
- **Head wave**
- **Diffracted wave**

  Will have the velocity close to the direct wave

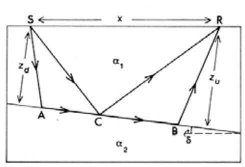
Reverse the line

- Shooting to the same string of geophones from the other end
- Two traveltime segments: direct and head wave

  Head wave generated from energy entering the high velocity layer at the vertical interface



Dipping layers



Dipping layers still produce head waves but the traveltimes are affected by the dip

Shooting up-dip: the velocity appears greater

Shooting down-dip: the velocity is reduced

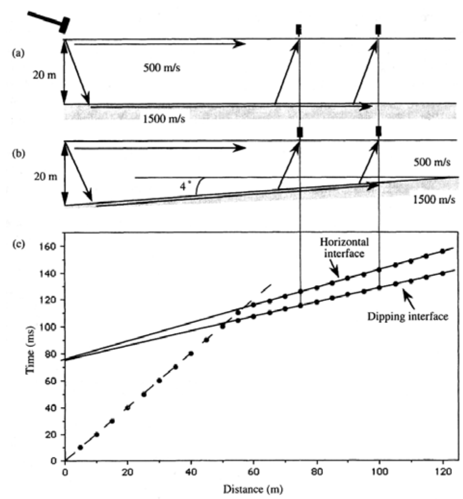


Figure 3-15 (a) Horizontal interface at a depth of 20 m with velocities above and below the interface of 500 m/s and 1500 m/s, respectively. (b) A dipping interface with identical depth to that in (a) at the site of the hammer impact and with identical velocities to (a). (c) Travel-time curves for (a) and (b).

Reversing lines

...shooting to a line of geophones from both ends

For horizontal layers the traveltime curves are symmetrical

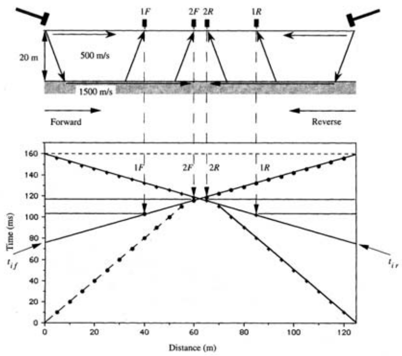


Figure 3-16 Correlation of a travel-time curve with wave paths to geophones at equal distances from an energy source for a forward and a reverse traverse.

For dipping layers layer velocities appear different for each end – the dip and true velocity can be determined from the up-dip and down-dip velocities

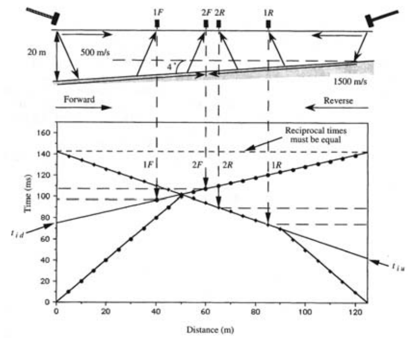
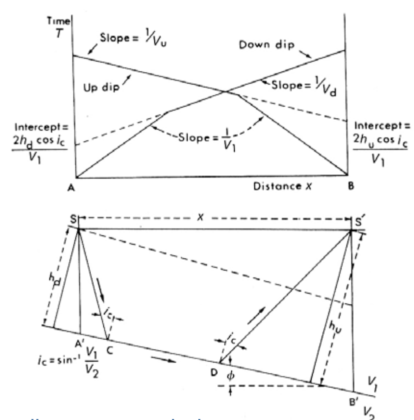


Figure 3-17 Correlation of a travel-time curve with geophone positions above a single dipping interface. The purpose of this diagram is to demonstrate the different path distances and arrival times for geophones located at identical offsets for a forward and reverse traverse.

Dipping layer traveltime



Down-dip

$$T_d = \frac{SC}{V_1} + \frac{CD}{V_2} + \frac{DS'}{V_1}$$

$$T_d = \frac{h_u + h_d}{V_1 \cos i_c} + \frac{x - \left[(h_u + h_d)\tan i_c\right]}{V_2}$$

With trigonometric transformations, an exercise for the class:

Down-dip traveltime

$$T_d = \frac{x \sin(i_c + \phi)}{V_1} + \frac{2h_d \cos i_c}{V_1}$$

Up-dip traveltime

$$T_u = \frac{x \sin(i_c - \phi)}{V_1} + \frac{2h_u \cos i_c}{V_1}$$

Down-dip apparent velocity

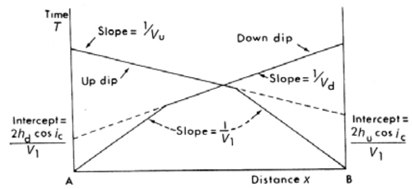
$$V_d = \frac{V_1}{\sin(i_c + \phi)}$$

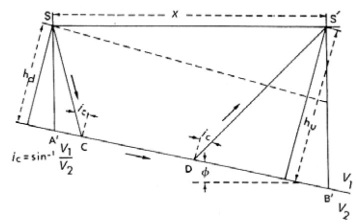
Up-dip apparent velocity

$$V_u = \frac{V_1}{\sin(i_c - \phi)}$$

...where is V_2 dependence?

Dipping layer traveltime





Given

$$V_d = \frac{V_1}{\sin(i_c + \phi)} \qquad V_u = \frac{V_1}{\sin(i_c - \phi)}$$

We can solve for:

$$\phi = \tfrac{1}{2}\left[\sin^{-1}\frac{V_1}{V_d} - \sin^{-1}\frac{V_1}{V_u}\right]$$

$$i_c = \tfrac{1}{2}\left[\sin^{-1}\frac{V_1}{V_d} + \sin^{-1}\frac{V_1}{V_u}\right]$$

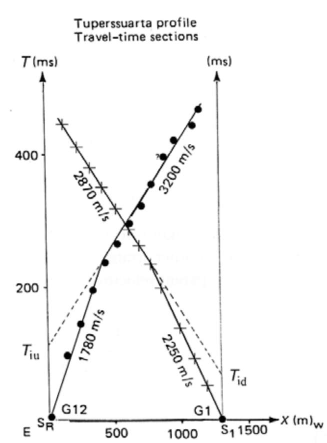
V_2 then obtained from: $\sin i_c = \dfrac{V_1}{V_2}$

Finally, the intercept times can be used to determine the perpendicular distance to the reflector:

$$T_{id} = \frac{2h_d \cos i_c}{V_1} \qquad T_{iu} = \frac{2h_u \cos i_c}{V_1}$$

Dipping layer

Example



Direct arrivals

Velocities from slopes: 1780 m/s and 2250 m/s

➔ average: 2015 m/s

Head waves

Up-dip velocity, V_u = 3200 m/s

Down-dip velocity, V_d = 2870 m/s

Using

$$\phi = \tfrac{1}{2}\left[\sin^{-1}\frac{V_1}{V_d} - \sin^{-1}\frac{V_1}{V_u}\right]$$

$$i_c = \tfrac{1}{2}\left[\sin^{-1}\frac{V_1}{V_d} + \sin^{-1}\frac{V_1}{V_u}\right]$$

we obtain:

ϕ = 2.8°

i_c = 42°

Dipping layer

Example

Now obtain V_2 from

$$\sin i_c = \frac{V_1}{V_2}$$

$\boldsymbol{V_2}$ **= 3000 m/s**

To determine the perpendicular depths, h_u and h_d, use

$$T_{id} = \frac{2h_d \cos i_c}{V_1} \qquad T_{iu} = \frac{2h_u \cos i_c}{V_1}$$

$\boldsymbol{h_u}$ **= 155 m** and $\boldsymbol{h_d}$ **= 95 m**