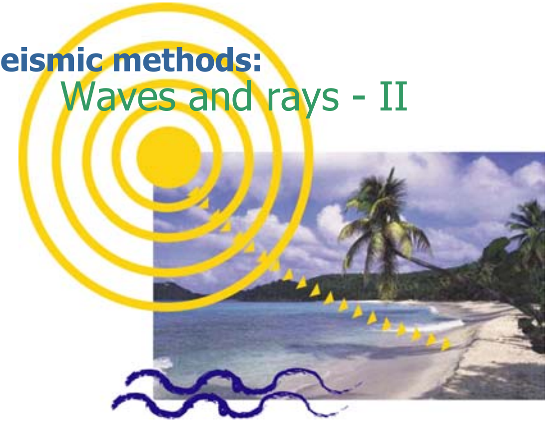
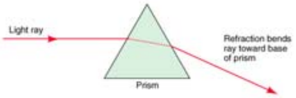
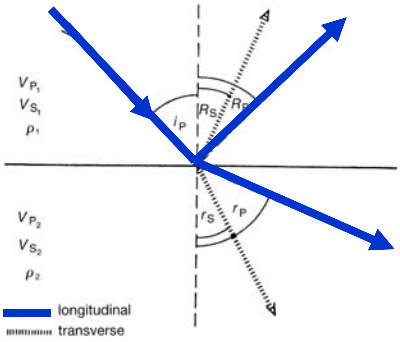
Seismic methods:
Waves and rays - II



Reading:
Today: p117-133
Next Lecture: p133-143

Reflection and transmission





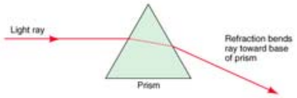
Seismic rays obey Snell's Law

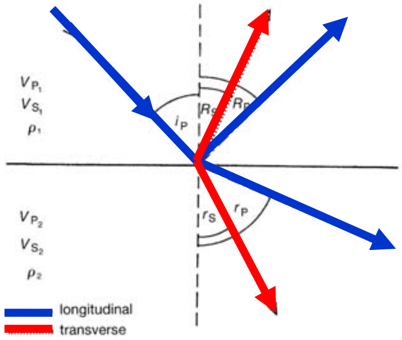
(just like in optics)

The angle of incidence equals the angle of reflection, and the angle of transmission is related to the angle of incidence through the velocity ratio.

$$\frac{\sin i_P}{V_{P1}} = \frac{\sin R_P}{V_{P1}} = \frac{\sin r_P}{V_{P2}}$$

Reflection and transmission





Seismic rays obey Snell's Law

(just like in optics)

The angle of incidence equals the angle of reflection, and the angle of transmission is related to the angle of incidence through the velocity ratio.

But a conversion from P to S or vice versa can also occur. Still, the angles are determined by the velocity ratios.

$$\frac{\sin i_P}{V_{P1}} = \frac{\sin R_P}{V_{P1}} = \frac{\sin r_P}{V_{P2}} = \frac{\sin R_S}{V_{S1}} = \frac{\sin r_S}{V_{S2}} = p$$

where p is the **ray parameter** and is constant along each ray.

Amplitudes reflected and transmitted

The amplitude of the reflected, transmitted and converted phases can be calculated as a function of the incidence angle using **Zoeppritz's equations**.

Simple case: **Normal incidence**

Reflection coefficient

$$R_C = \frac{A_R}{A_i} = \frac{\rho_2 V_2 - \rho_1 V_1}{\rho_2 V_2 + \rho_1 V_1}$$

Transmission coefficient

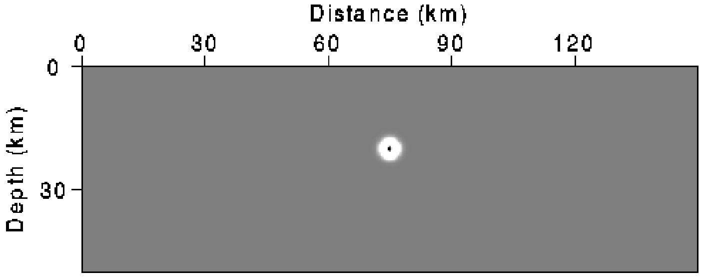
$$T_C = \frac{A_T}{A_i} = 1 - R_C = \frac{2\rho_1 V_1}{\rho_2 V_2 + \rho_1 V_1}$$

Reflection and transmission coefficients for a specific impedance contrast

These coefficients are determined by from the product of velocity and density – the **impedance** of the material.

R_C usually small – typically 1% of energy is reflected.

Reflection and transmission



P-wave Propagation

You can see:
a direct wave, reflected and transmitted waves, plus multiples…

Normal move out (NMO)

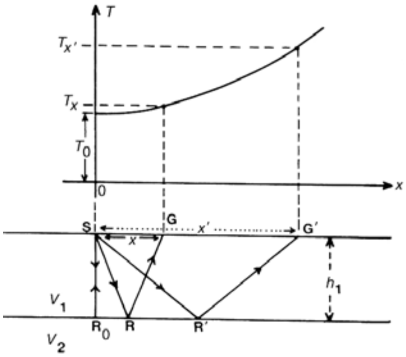
Reflection from a single horizontal impedance contrast:

Arrival time

$$T_x = \frac{2SR}{V_1} = \frac{2}{V_1}\sqrt{h_1^2 + \left(x/2\right)^2}$$

or

$$T_x^2 = T_0^2 + \frac{x^2}{V_1}$$



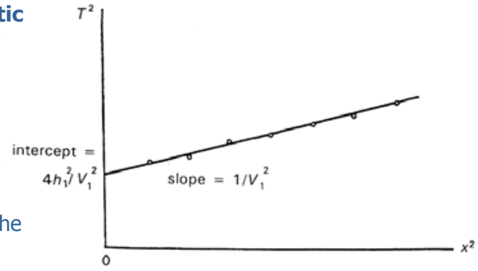
The arrival time curve is a hyperbola

Note: a geophone spread GG’ samples RR’ of the reflector. RR’=GG’/2

Normal move out (NMO)

Arrival time curve is quadratic

$$T_x^2 = T_0^2 + \frac{x^2}{V_1^2}$$



So, if plot T^2 vs. x^2 we can determine the V_1 and h_1 from the slope and intercept

The importance of NMO

- Having determined the layer velocity, we can use the predicted quadratic shape to identify reflectors
- Then correct (shift traces) and stack to enhance signal to noise

$$\Delta T_{NMO} = T_x - T_0 \approx \frac{x^2}{2T_0 V_1^2}$$

Multiple layers

Use Snell's Law to trace ray paths

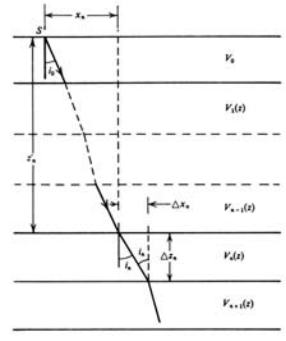
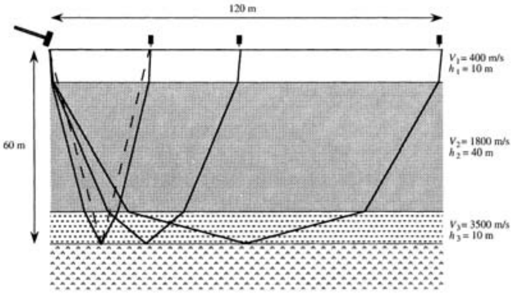


Figure 4.29. Raypath where velocity varies with depth.



At each interface

$$\frac{\sin i_P}{V_{P1}} = \frac{\sin r_P}{V_{P2}} = p$$

NMO for layers

When the offset is small w.r.t. reflector depth (x<<h), the NMO curve is still a hyperbola

$$T^2 = T_{0,n}^2 + \frac{x^2}{\overline{V}_{rms,n}^2}$$

where

$$T_{0,n} = \sum_{k=1}^{n} T_{0,k} = \sum_{k=1}^{n} \frac{2h_k}{V_k}$$

$$\overline{V}_{rms,n}^2 = \frac{\sum_{k=1}^{n} V_k^2 T_k}{\sum_{k=1}^{n} T_k}$$

Determine velocity structure one layer at a time

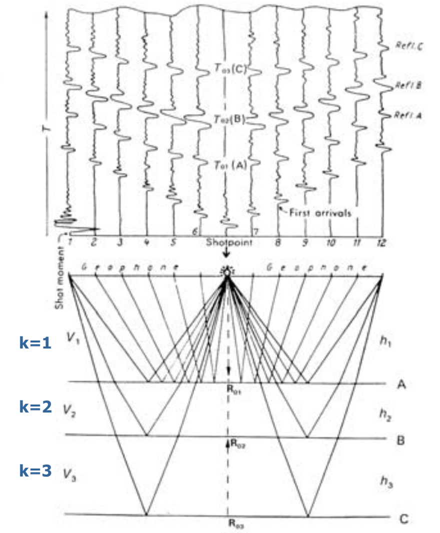
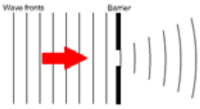


Fig. 4.14 Schematic diagram showing the production of a reflection seismogram. The 12-trace record shows the time sequence of the reflected pulses from reflecting horizons, A, B, and C. T_{01}, T_{02}, and T_{03} are the two-way vertical travel-times from points R_{01}, R_{02}, and R_{03}, respectively, below the shotpoint. The significance of first arrivals is discussed in Section 4.4.5.

Diffraction

From optics



A simple demonstration of diffraction

A sharp break in a reflector acts as a secondary source of a spherical wavefront

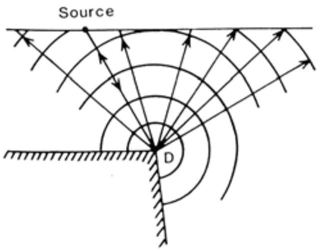
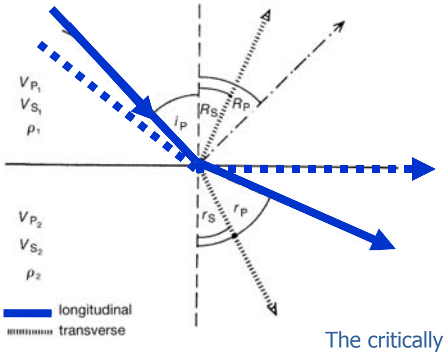


Fig. 4.5 Diffracted wavefronts from a sharp edge, D, which has been set into oscillation by waves coming from a seismic source.

Critical incidence



$$\frac{\sin i_P}{V_{P1}} = \frac{\sin r_P}{V_{P2}}$$

when $V_2 > V_1$, $r_P > i_P$

therefore, we can increase i_P until $r_P = 90°$

When $r_P = 90°$ $i_P = i_C$ the critical angle

$$\sin i_C = \frac{V_{P1}}{V_{P2}}$$

The critically refracted energy travels along the velocity interface at V_2 continually refracting energy back into the upper medium at an angle i_C

➔ a **head wave**

Head wave

- Occurs due to a low to high velocity interface
- Energy travels along the boundary at the higher velocity
- Energy is continually refracted back into the upper medium at an angle i_C
- Provides constraints on the boundary depth e.g. Moho depth

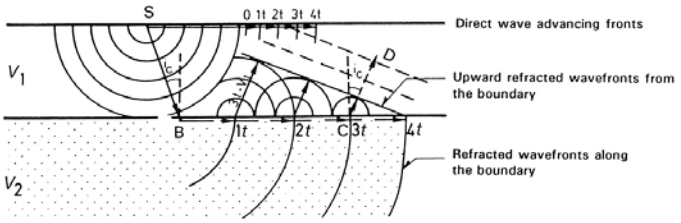
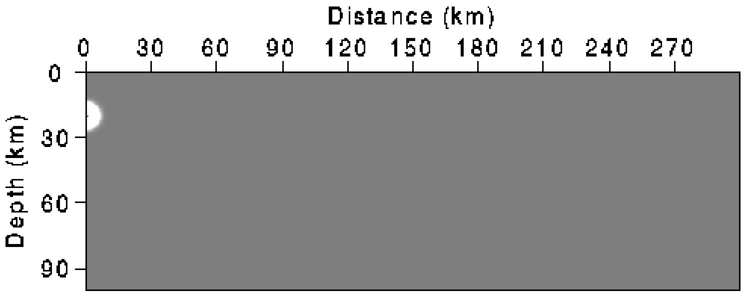


Fig. 4.6 Schematic illustration showing the ray paths of the incident wave (SB) striking the boundary at critical angle (i_c), and the refracted wave (BC) traveling along the boundary with velocity V_2 ($>V_1$). The latter is refracted back to the first medium (V_1) at the same angle (i_c) and re-emerges with a ray path such as CD. Advancement of the wavefronts is shown from the instant ($t=0$) when the incident ray strikes the boundary at B. (Modified from Klitten, 1987.)

Head wave



Reflection and Refraction at a Boundary

You can see:

a head wave, trapped surface wave, diving body wave

Factors affecting velocity

$$V_P = \sqrt{\frac{\kappa + \frac{4}{3}\mu}{\rho}}$$

$$V_S = \sqrt{\frac{\mu}{\rho}}$$

Density – velocity typically increases with density

(κ and μ are dependant on ρ and increase more rapidly than ρ)

Porosity and **fluid saturation**

Increasing porosity reduces velocity.

Filling the porosity with fluid increases the velocity.

$$\frac{1}{V_{sat}} = \frac{\phi}{V_F} + \frac{1-\phi}{V_M}$$

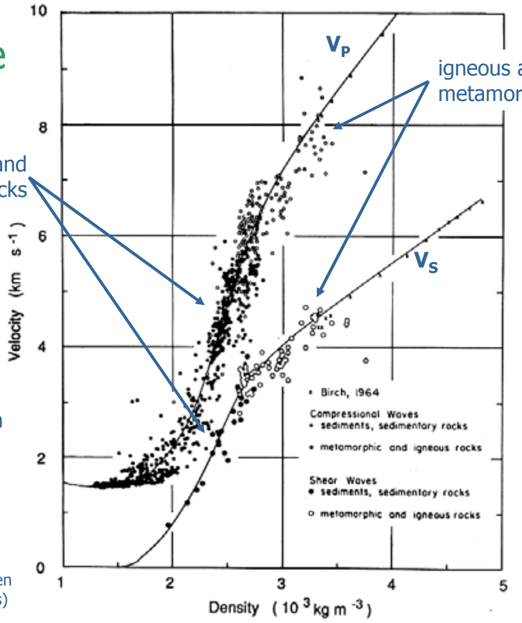
Poisson's ratio – related to V_P/V_S

This is used to distinguish between rock/sediment types. It is usually more sensitive than just V_P alone.

The significant variations in sediments are usually due to porosity variations and water saturation. Water saturation has no effect on V_S (for low porosities) but a significant effect on V_P.

Velocity and density

Nafe-Drake curve



V_P

igneous and metamorphic rocks

sediments and sedimentary rocks

V_S

This curve has been approximated using the expression

$$\rho = aV_P^{1/4}$$

(a is a constant: 1670 when ρ in km/m^3 and V_P in km/s)

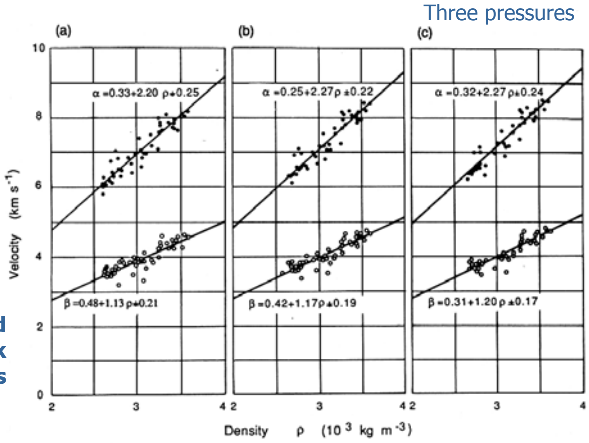
Velocity and density

Birch's Law

A linear relationship between velocity and density

v = aρ + b

Three pressures



Crust and mantle rock observations

Typical rock velocity ranges

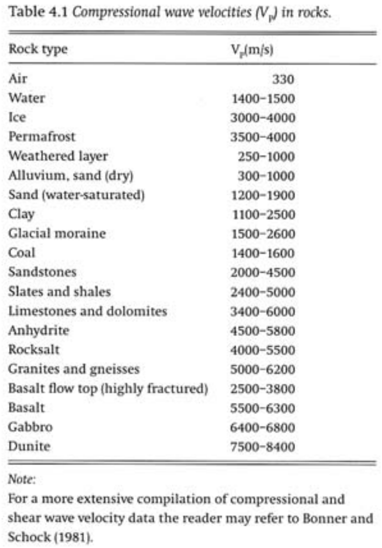


Table 4.1 *Compressional wave velocities (V_p) in rocks.*

Rock type	V_p(m/s)
Air	330
Water	1400–1500
Ice	3000–4000
Permafrost	3500–4000
Weathered layer	250–1000
Alluvium, sand (dry)	300–1000
Sand (water-saturated)	1200–1900
Clay	1100–2500
Glacial moraine	1500–2600
Coal	1400–1600
Sandstones	2000–4500
Slates and shales	2400–5000
Limestones and dolomites	3400–6000
Anhydrite	4500–5800
Rocksalt	4000–5500
Granites and gneisses	5000–6200
Basalt flow top (highly fractured)	2500–3800
Basalt	5500–6300
Gabbro	6400–6800
Dunite	7500–8400

Note:
For a more extensive compilation of compressional and shear wave velocity data the reader may refer to Bonner and Schock (1981).

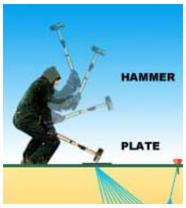
Using velocity alone to determine rock type is problematic to impossible.

Seismic sources

Consider
- **Energy input**
- **Repeatability**
- **Cost**
- **Convenience**

Rifles and guns
- Cheap
- Repeatable – fire into water filled hole
- Shallow targets 0-50m

Sledge hammer
- Cheap
- Repeatable once plate is stable (and with training!)
- Targets 15-50m



Weight drops
- Cheap
- Repeatable – automated
- Targets > 50m

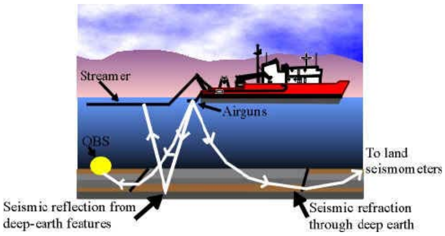
Seismic sources

Consider
- **Energy input**
- **Repeatability**
- **Cost**
- **Convenience**

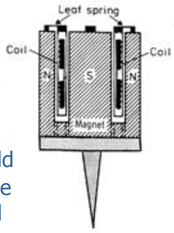
Vibroseis
- No pulse, frequency sweep
- Significant signal with stacking/deconvolution

Explosives
- Various sizes – target depth
- Safety and expense can be an issue

Air guns
- At sea
- Very repeatable
- Large array for big signal



Seismic receivers



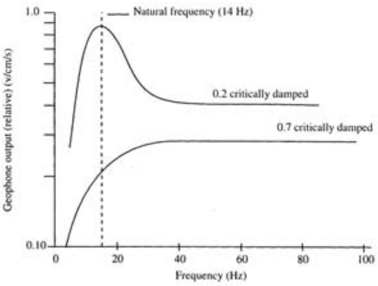
Geophones
- Cylindrical coil suspended in a magnetic field
- The inertia of the coil causes motion relative to the magnet generating a electrical signal
- Geophones are sensitive to velocity

Hydrophones
- Used at sea
- Use piezoelectric minerals to sense pressure variations

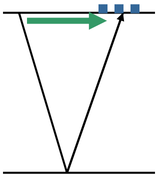
Instrument response
- The relation between the input ground motion and the output electrical signal

Natural frequency
- The frequency which produces the maximum amplitude output

Damping
- Reduces the amplitude of the natural frequency response and prevents infinite oscillations
- Want a **flat response**



Deployment



Important considerations

- Need good coupling to the ground – spike
- Mini-arrays to reduce surface wave noise

Offset of geophones

Small offsets

- Near-vertical incidence retains P-energy
- High resolution of subsurface reflectors

➔ **Seismic reflection analysis**

Large offsets

- Improves velocity sensitivity
- Provides horizontal averages only

➔ **Seismic refraction analysis**