Gravity:
Theory and measurement

Reading:
Today: p11 - 22

Theory of gravity

Use two of Newton's laws:

1) Universal law of gravitation: $F = \frac{G m_1 m_2}{r^2}$

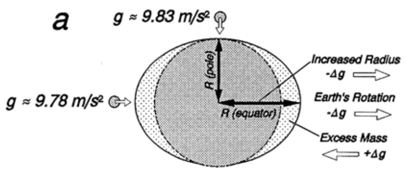
2) Second law of motion: $F = mg$

We can combine them to obtain the gravitational acceleration at the surface of the earth:

$$g = \frac{GM_E}{R_E^{\;2}}$$

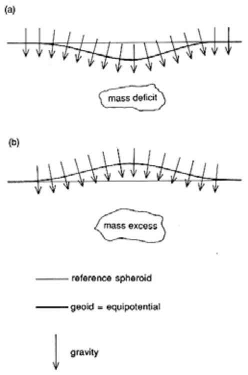
Is the Earth's gravitational acceleration a constant?

Variations in g



Large scale variations: global or regions

Figure 5.8. (a) A trough in the geoid, or negative geoid height anomaly, occurs over a region of mass deficit (such as a depression in the seabed). A negative free-air gravity anomaly also occurs over such a mass deficit. (b) A bulge in the geoid, or positive geoid height anomaly, occurs over regions of excess mass (such an elevated region of the seabed). A positive free-air gravity anomaly also occurs over such a mass excess.



Smaller scale variations: local

This is what we want to make use of

The geoid

Mean sea level is an equipotential surface
➔ it is the geoid

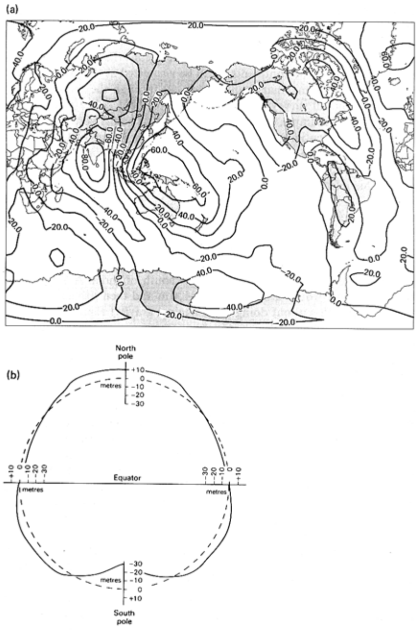


Figure 5.3. (a) Geoid height anomalies: the height of the geoid above (positive) or below (negative) the spheroid in metres. (b) The averaged shape of the earth, calculated by assuming that the earth is symmetric about its rotation axis (solid line), compared with a spheroid of flattening 1/298.25 (dashed line). (From King-Hele 1969.)

Gravity and potentials

g is a vector field:

$$\mathbf{g} = \frac{GM_E}{R_E^{\ 2}}\mathbf{r}_1$$

where r_1 is the unit vector pointing toward the center of the Earth

Gravitational potential:

$$U = \frac{Gm}{r}$$

U is a scalar field which makes it easier to work with

Definition: The gravitational potential, U, due to a point mass m, at a distance r from m, is the work done by the gravitational force in moving a unit mass from infinity to to a position r from m.

Relating *g* to *U*

U is a scalar field which makes it easier to work with:

- **Potentials are additive**
- **Gravity is a conservative force**
- **And gravitational acceleration can be easily determined from the potential…**

Given: $U = \frac{Gm}{r}$

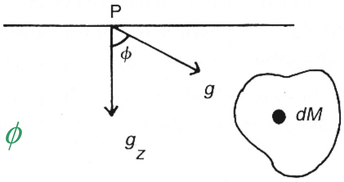
It follows that:

$$\mathbf{g} = -\frac{\partial U}{\partial r} = \frac{Gm}{r^2}$$

For smaller scale problems we usually deal with g, and sum the vertical component of g…

Gravity anomalies

Sum contributions in the vertical direction



$$g_z = G\int_M \frac{dM}{r^2}\cos\phi = G\int_V \frac{\rho dV}{r^2}\cos\phi$$

Or, in Cartesian coordinates:

$$g_z = G\iiint \frac{\rho z dx dy dz}{r^3}$$

where

$$r = \sqrt{(x-\alpha)^2 + (y-\beta)^2 + z^2}$$

This is ideal for implementation in a computer code.

Units for g

SI unit for g: m/s² – though you will rarely see this!

1 cm/s² = 1 Gal (for Galileo) = 0.01 m/s²

milliGal or mGal = 10^{-3} Gal – typical unit for field studies

Our text book uses the "gravity unit" (g.u.)

1 g.u. = 0.1 mGal

Normal value of g at the surface of the Earth:

g_E = 9.8 m/s² = 980 cm/s² = 980 Gal = 980,000 mGal = 9800 g.u.

Rock density

Mass = Density x Volume

Lateral variations in rock density result in gravity anomalies that can be measured at the surface

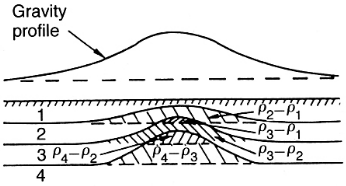


Fig. 2.2 Schematic section showing lateral density contrasts resulting from a structural uplift. ρ_1, ρ_2, ρ_3, and ρ_4 are the densities of four flat-lying layers. Horizontal variation in gravity is caused by lateral variations in density. (After Nettleton, 1971.)

Factors influencing rock density

Unconsolidated sediments – composition, porosity, saturation

Sedimentary rocks – composition, age and depth of burial (compaction), cementation, porosity, pore fluid

Igneous rocks – composition (esp. silica content), crystal size, fracturing (i.e. porosity)

Metamorphic rocks – composition (esp. silica content), metamorphic grade, fracturing (i.e. porosity)

Porosity and **pore fluid content** are probably the most important factors affecting density in the shallow sub-surface

Table of rock densities

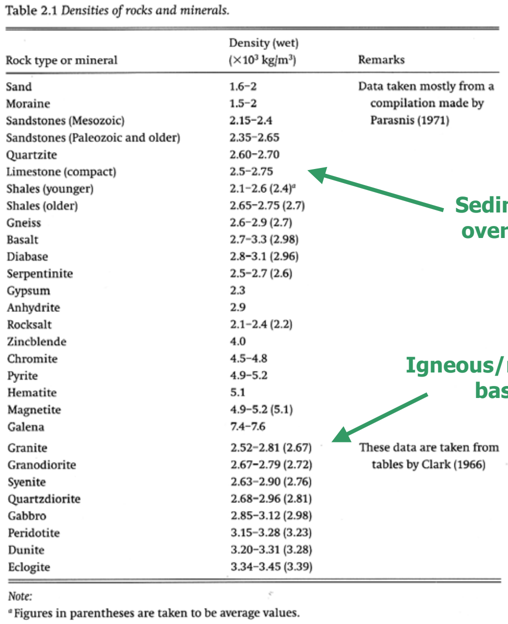


Table 2.1 *Densities of rocks and minerals.*

Rock type or mineral	Density (wet) ($\times 10^3$ kg/m^3)	Remarks
Sand	1.6–2	Data taken mostly from a compilation made by Parasnis (1971)
Moraine	1.5–2	
Sandstones (Mesozoic)	2.15–2.4	
Sandstones (Paleozoic and older)	2.35–2.65	
Quartzite	2.60–2.70	
Limestone (compact)	2.5–2.75	
Shales (younger)	2.1–2.6 (2.4)[a]	
Shales (older)	2.65–2.75 (2.7)	
Gneiss	2.6–2.9 (2.7)	
Basalt	2.7–3.3 (2.98)	
Diabase	2.8–3.1 (2.96)	
Serpentinite	2.5–2.7 (2.6)	
Gypsum	2.3	
Anhydrite	2.9	
Rocksalt	2.1–2.4 (2.2)	
Zincblende	4.0	
Chromite	4.5–4.8	
Pyrite	4.9–5.2	
Hematite	5.1	
Magnetite	4.9–5.2 (5.1)	
Galena	7.4–7.6	
Granite	2.52–2.81 (2.67)	These data are taken from tables by Clark (1966)
Granodiorite	2.67–2.79 (2.72)	
Syenite	2.63–2.90 (2.76)	
Quartzdiorite	2.68–2.96 (2.81)	
Gabbro	2.85–3.12 (2.98)	
Peridotite	3.15–3.28 (3.23)	
Dunite	3.20–3.31 (3.28)	
Eclogite	3.34–3.45 (3.39)	

Note:

[a] Figures in parentheses are taken to be average values.

Sedimentary overburden

Igneous/metamorphic basement

Similarity in rock densities can make it difficult to distinguish

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Measuring g:

Absolute and relative

- g at the Earth's surface ~ 980,000 mGal
- variations in g on the order 1 mGal

➔ need to measure g to better than 1 part in 1 million

➔ use instruments sensitive to relative changes in g

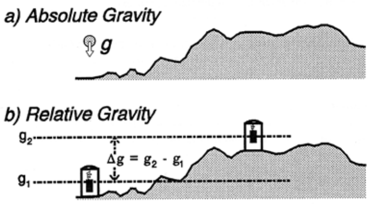


FIGURE 8.13 a) Absolute gravity is the true gravitational acceleration (g). b) Relative gravity reflects the difference in gravitational acceleration (Δg) at one station (g_1) compared to another (g_2).

Measuring g:

Absolute gravity

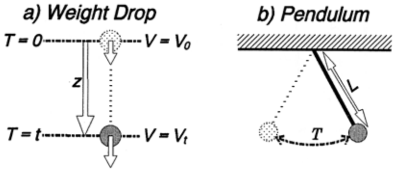
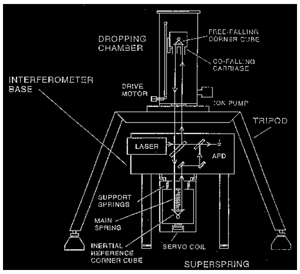


FIGURE 8.14 *Measurement of absolute gravity.* a) *Weight drop.* The object accelerates from an initial velocity of V_0 at time (T = 0), to a velocity of V_t at time (T = t), as it falls a distance (z). b) *Pendulum.* Gravitational acceleration is a function of the pendulum's length (L) and period of oscillation (T).



Measuring g:

Stable gravimeter

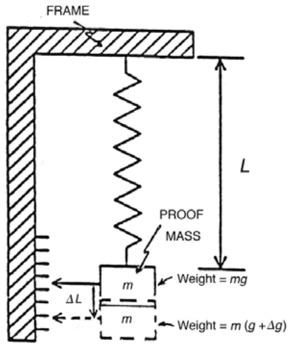


Fig. 2.3 Principle of operation of a stable gravimeter. To measure gravity changes to 0.1 g.u. (i.e., to about 1 part in 10^8 of the earth's normal gravity) would require the fractional change in length $\Delta L/L$ to be measured to a precision of the same order.

change in g ➔ change in spring length

Hooke's Law $\Delta F = -k\,\Delta L$

and $\Delta g = -k\,\Delta L/m$

if $\Delta g/g = 10^{-6}$

then $\Delta L/L = 10^{-6}$

This requires high optical, mechanical or electronic magnification

Measuring g:

Unstable gravimeter

Applies and additional negative restoring force to amplify changes in g

Uses a zero length spring: the restoring force is equal to the length of the spring

Suitable choice of mass, spring constant and geometry makes the system unstable and very sensitive to changes in g

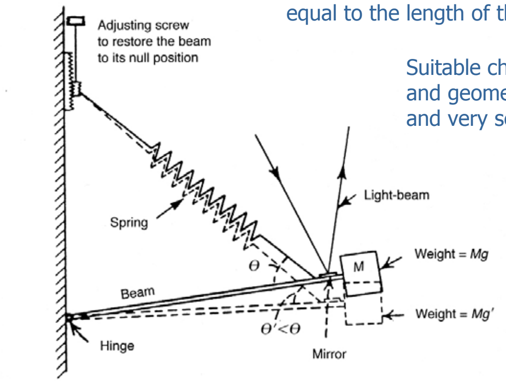
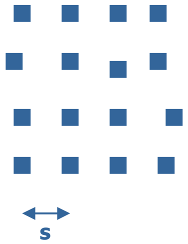


Fig. 2.4 Constructional design and basic components of the LaCoste–Romberg gravimeter.

LaCoste-Romberg gravimeter

Gravity surveying

Survey design



Survey design considerations

- **Uniform grid – for easier interpretation**
- **Station spacing: s < h**

  h is the depth of the body of interest
- **Avoid steep tomographic gradients**
- **Absolute and relative station locations are needed** …how accurate?

Typical station spacing

Regional geologic studies: km to 10s of km

Local structure/Engineering/Environmental: 10s to 100s m

Near surface e.g. archeology: few meters

Gravity surveying
Drift

The reading of a gravimeters at a point changes with time!

Causes

- **Instrument drift:** due to environmental changes (P,T) and spring creep
- **Earth tides:** relative rotations of the earth, moon and sun

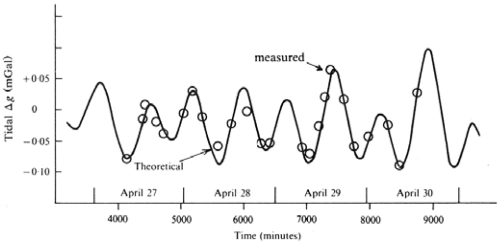


Figure 2.6. Earth-tide variations, Montreal, April 1969. Gravity readings have been corrected for instrument drift.

Gravity surveying
Correcting for drift

1. **Return to base station periodically**
2. **Assume drift is linear**
3. **Correct measurements in loop**

How often?

Depends on requires accuracy

- max tidal rate: 0.05 mGal/hr
- instrument drift usually less